1. (5 points) Use Green’s theorem to evaluate the line integral \( \int_C y^3 dx - x^3 dy \), where \( C \) is the circle \( x^2 + y^2 = 4 \), positively oriented. Explain your work.

Solution: Let \( D \) be the enclosed domain. Then \( D = \{(x,y), x^2 + y^2 \leq 4\} \). Since \( C = \partial D \) is positively oriented, applying Green’s theorem gives

\[
I = \int_C y^3 dx - x^3 dy = \iint_D -3x^2 - 3y^2 dA = -3 \int_D x^2 + y^2 dA.
\]

Passing onto polar coordinates the domain is \( \{(r,\theta), 0 \leq r \leq 2, 0 \leq \theta \leq 2\pi\} \). It gives

\[
I = -3 \int_0^{2\pi} \int_0^2 r^2 r dr d\theta = -6\pi \int_0^2 r^3 dr = -6\pi \left[\frac{r^4}{4}\right]_0^2 = -24\pi.
\]

2. (5 points) We consider the vector field \( \mathbf{F}(x,y,z) = <e^x \sin y, e^y \sin z, e^z \sin x> \). Give the curl and divergence of the vector field.

Solution: Simple computation.