Math 2263 - Quiz 2
Planes, Distances, Surfaces

Name: __________________________ Score: __________________________

This is a 20 minutes quiz, one-sided. No calculators, computers, cellphones, notes, book allowed. Show all work. No credit will be given for answers without work.

1. Let $P$ be the plane with equation $x = 1$.
   (a) (2 points) Recall the formula of the distance between $P$ and a point $Q(x_0, y_0, z_0)$, denoted $\text{dist}(Q, P)$.

   **Solution:** The plane $P$ has equation $x - 1 = 0$. We apply the formula:
   \[
   \text{dist}(Q, P) = \frac{|x_0 - 1|}{\sqrt{1^2}} = |x_0 - 1|.
   \]

   (b) (5 points) We consider the point $A(-1, 0, 0)$. We say that $Q$ is equidistant from $P$ and $A$ if $\text{dist}(Q, P) = |AQ|$. Find an equation of the surface consisting of all points $Q$ equidistant from $P$ and $A$. Identify the surface.

   **Solution:** Let $S$ be the surface consisting of all points $Q(x, y, z)$ equidistant from $P$ and $A$. Then, $|AQ| = \sqrt{(x + 1)^2 + y^2 + z^2}$ and $\text{dist}(Q, P) = |x - 1|$. Hence the coordinates $(x, y, z)$ satisfy
   \[
   |x - 1| = \sqrt{(x + 1)^2 + y^2 + z^2}
   \]
   \[
   (x - 1)^2 = (x + 1)^2 + y^2 + z^2
   \]
   which gives
   \[
   4x + y^2 + z^2 = 0
   \]
   for the equation of $S$. It is a circular paraboloid, along the negative $x$–axis.

2. (a) (3 points) Find the point at which the line $L$ whose parametric equations are $x = t - 1, y = 1 + 2t, z = 3 - t$ intersects the plane $P$ given by $3x - y + 2z = 5$.

   **Solution:** Let $Q(x_0, y_0, z_0)$ be the intersection point, if it exists. As $Q$ belongs to $L$ there exists $t_0$ such that $x_0 = t_0 - 1, y_0 = 1 + 2t_0, z_0 = 3 - t_0$. As $Q$ belongs to $P$, $t_0$ must satisfy the equation
   \[
   3(t_0 - 1) - (1 + 2t_0) + 2(3 - t_0) = 5,
   \]
   which gives $t_0 = -3$. Hence, they intersect at the point $Q(-4, -5, 6)$.

   (b) (1 point) What is the distance between the line and the plane?

   **Solution:** They intersect, so the distance is 0.