1. (5 points) Compute \( I = \iiint_E \sqrt{x^2 + y^2} \, dV \) where \( E \) is the region that lies inside the cylinder \( x^2 + y^2 = 16 \) and between the planes \( z = -5 \) and \( z = 4 \).

Solution:

We describe the domain in cylindrical coordinates. It yields:

\[
E = \{(r, \theta, z), \ r \in [0, 4], \ \theta \in [0, 2\pi), \ z \in [-4, 5]\}.
\]

Hence the integral reads

\[
I = \iiint_E \sqrt{x^2 + y^2} \, dV = \int_0^{2\pi} \int_0^4 \int_{-4}^5 r \, dz \, r \, dr \, d\theta = 18\pi \int_0^4 r^2 \, dr = 18\pi \left[ \frac{r^3}{3} \right]_0^4 = 384\pi.
\]

2. (6 points) Compute \( I = \iiint_E x \, dV \) where \( E \) is bounded by the paraboloid \( x = 4y^2 + 4z^2 \) and the plane \( x = 4 \).

Solution:

The region \( E \) can be describe as a simple region, with \( x \) bounded between two surfaces, graphs of functions of \( y \) and \( z \):

\[
E = \{(x, y, z), \ (y, Z) \in D, \ 4y^2 + 4z^2 \leq x \leq 4\}.
\]

The domain \( D \) in the \( yz \)-plane is the projection of the region \( E \) onto the \( yz \)-plane, here the disk centered at 0 of radius 1.

Hence we can write

\[
I = \iiint_E x \, dV = \int_D \int_{4y^2+4z^2}^4 x \, dx \, dA = \int_D \frac{1}{2} \left( 16 - (4y^2 + 4z^2)^2 \right) dA.
\]

The domain \( D \) can easily be described in polar coordinates in the \( yz \)-plane. With \( y = r \cos \theta \) and \( z = r \sin \theta \), we have \( D = \{(r, \theta), \ \theta \in [0, 2\pi), \ r \in [0, 1]\} \). Thus

\[
I = \int_0^{2\pi} \int_0^1 \frac{1}{2} (16 - (4r^2)^2) r \, dr \, d\theta = \pi \int_0^1 16r - 16r^5 \, dr = \frac{16\pi}{3}.
\]