• This is a 50 minutes exam.

• No books, cell phones, calculators, etc. are allowed. One page of formula sheet, one sided, hand written, is allowed.

• This exams contains 6 pages (including this cover page), plus 2 pages of scratch paper, and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

• Do not give numerical approximations to quantities such as \( \sin(5) \), \( \pi \), or \( \sqrt{2} \). However, you should simplify \( \cos\left(\frac{\pi}{2}\right) = 0 \), \( e^0 = 1 \), and so on.

The following rules apply:

• **Show your work, in a reasonably neat and coherent way**, in the space provided. **All answers must be justified** by valid mathematical reasoning supported by the results seen in class.

• A correct answer, unsupported by calculations, explanations, and/or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

• Please circle your answers and underline the key steps in your explanations and computations.

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1. (20 points) We consider the following matrix depending on the real parameter $k$:

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & k & -1 \\ 1 & k^2 & 1 \end{pmatrix}.$$

What are the values of the parameter $k$ such that the matrix $A$ is invertible?
2. (20 points) Find the least square solution (or solutions) of the equation

\[ A \vec{x} = \vec{b} \]

where \( A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \) and \( \vec{b} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} \).
3. (20 points) We consider the following matrix \( A = \begin{pmatrix} -5 & 2 & 3 \\ -7 & 4 & 3 \\ -8 & 2 & 6 \end{pmatrix} \).

(a) (15 points) Determine if the matrix \( A \) is diagonalizable, and if it is give a diagonal matrix similar to \( A \) (you do NOT have to find the change of basis matrix).

(b) (5 points) What is the rank of the matrix \( A \)?
4. (20 points) Let $V \subset \mathbb{R}^3$ be the linear subspace given by

$$V = \text{Span} \left\{ \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}, \begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix} \right\}.$$

Find the orthogonal projection of the vector $\begin{pmatrix} 49 \\ 49 \\ 49 \end{pmatrix}$ onto the subspace $V$. 
5. (20 points) Let $V \subset \mathbb{R}^4$ be the solution space of the linear system

\[
\begin{align*}
    x_1 - x_2 + x_3 - x_4 &= 0 \\
    x_1 + 3x_2 + x_3 - 5x_4 &= 0.
\end{align*}
\]

Find a basis of $V^\perp$, the orthogonal of $V$. 

Scratch paper