Homework 6

MATH 8652 Spring 2016

Q1. If $(B_t)_{t\geq 0}$ is a standard BM, show that, as $t \to \infty$,

$$\left(\int_0^t e^{B_s} ds\right)^{1/\sqrt{t}} \xrightarrow{d} e^{M_1},$$

where $M_1 = \sup_{0\leq s\leq 1} B_s$.

Q2. Let $(B_t)_{t\geq 0}$ be a standard BM and let $R = \inf\{t \geq 1 : B_t = 0\}$ and $L = \sup\{t \leq 1 : B_t = 0\}$.

(a) Using Markov property at time 1, show that $R \sim 1 + \frac{\xi_1^2}{\xi_2^2}$ where ξ_1 and ξ_2 are two independent standard normals.

(b) Show that $R = L^{-1}$ in distribution and recover the arcsine law for L.

Q3-Q6. Durrett 8.1.3, 8.2.3, 8.2.4, 8.5.2.