1. (4 pt) Find the Jacobian of the transformation: \( x = 4u + v, \ y = 2u - v. \)

**Solution:** The Jacobian equals to \( \frac{\partial x}{\partial u} \frac{\partial y}{\partial v} - \frac{\partial x}{\partial v} \frac{\partial y}{\partial u} = 4 \times (-1) - 1 \times 2 = -6. \)

2. (5 pt) Find the image of the set \( S \) under the given transformation:

\( S \) is the square bounded by the line \( u = 0, \ u = 1, \ v = 0, \ v = 1; \ x = v, \ y = uv. \)

**Solution:** The transformation maps the boundary to the boundary. \( u = 0 \) will be mapped to \( y = 0, \ v = 0 \) will be mapped to the point \( (0, 0), \ v = 1 \) will be mapped to \( x = 1. \) For \( u = 1, \) the image will be \( x = v, \ y = v, \) which is just the line \( x = y. \) So the image of \( S \) under the transformation is just the triangular region bounded by \( x = 1, y = 0 \) and \( x = y. \)

3. (6 pt) Use the given transformation to evaluate the integral \( \iint_R x^2 \, dA \) where \( R \) is the region bounded by the ellipse \( 9x^2 + 4y^2 = 36; \ x = 2u, \ y = 3v. \)

**Solution:** The Jacobian of the transformation is \( 2 \times 3 = 6. \) Under this transformation, the region \( R \) will be mapped to \( S = \{(u, v) \mid 36u^2 + 36v^2 = 36\} = \{(u, v) \mid u^2 + v^2 = 1\}, \) which is just the region enclosed by unit circle. Our integral is equal to \( \iint_S 6(2u)^2 \, dA = \iint_S 24u^2 \, dA. \)

Use the polar coordinates, \( u = r \cos(\theta), \ v = r \sin(\theta), \) then \( r \in [0, 1], \ \theta \in [0, 2\pi]. \)

The integral will be \( \int_0^{2\pi} \int_0^1 24r^3 \cos^2(\theta) \, dr \, d\theta = \int_0^{2\pi} \frac{6 \cos^2(\theta)}{2} \, d\theta = \int_0^{2\pi} 3 \cos(2\theta) + 3d\theta = 6\pi. \)