1. (10 points) Reduce the equation

\[ 4x^2 + y^2 + 4z^2 - 4y - 24z + 36 = 0 \]

to one of the standard forms and state which kind of quadric surface it represents.

Solution: Completing squares, we get

\[
0 = 4x^2 + (y^2 - 4y + 4 - 4) + 4(z^2 - 6z + 9 - 9) + 36
\]
\[
= 4x^2 + (y - 2)^2 - 4 + 4(z - 3)^2 - 36 + 36.
\]

Rewriting,

\[
4x^2 + (y - 2)^2 + 4(z - 3)^2 = 4.
\]

Finally we divide by 4 to get the standard form

\[
x^2 + \frac{(y - 2)^2}{4} + (z - 3)^2 = 1,
\]

which is an ellipsoid.

2. (10 points) Show that the limit

\[
\lim \limits_{(x,y) \to (0,0)} \frac{y^3 \sin^2 x}{xy^4 + x^5}
\]
does not exist. Explicitly state along which paths you are evaluating the limit.

Solution: Approaching (0,0) along the x-axis y = 0, the numerator vanishes (while the denominator is nonzero), so the limit is zero.

Approaching (0,0) along the line y = x, the limit becomes

\[
\lim \limits_{x \to 0} \frac{x^3 \sin^2 x}{2x^5} = \lim \limits_{x \to 0} \frac{\sin^2 x}{2x^2} = \frac{1}{2} \lim \limits_{x \to 0} \left( \frac{\sin x}{x} \right)^2 = \frac{1}{2}
\]

because \( \lim_{x \to 0} \frac{\sin x}{x} = 1 \).

Since \( \frac{1}{2} \neq 0 \), the limit does not exist.
3. (10 points) Find the domain of the function $G(x, y) = 4 + \sqrt{25 - x^2}$ (in the form \{(x, y) : \ldots\}) and then sketch the domain in the $xy$-plane.

**Solution:** The only restriction is imposed by the square root, so the domain is
\[
\{(x, y) : 25 - x^2 \geq 0\} = \{(x, y) : x^2 \leq 25\} = \{(x, y) : -5 \leq x \leq 5\},
\]
which is a vertical strip on the $xy$-plane.