1. (4 points) The joint density function for a pair of random variables $X$ and $Y$ is

$$f(x, y) = \begin{cases} 
4xy & \text{if } 0 \leq x \leq 1, 0 \leq y \leq 1, \\
0 & \text{otherwise}.
\end{cases}$$

Find the expected value of $X$.

$$\iint_R x f(x, y) \, dA = \int_0^1 \int_0^1 4xy \, dx \, dy$$

$$= \int_0^1 \left[ 2x^2y \right]_0^1 \, dy$$

$$= 4 \int_0^1 y \, dy = \frac{4}{3} \cdot \frac{1}{2} = \frac{2}{3}.$$

2. (5 points) Write down an integral in polar coordinates that yields the surface area of the part of the paraboloid $z = 9 - x^2 - y^2$ that lies above the $xy$-plane. Do not evaluate the integral.

$$\text{Ans!} \quad I = \iint_D \sqrt{1 + f_x^2 + f_y^2} \, dA$$

where $f(x, y) = 9 - x^2 - y^2$ and $D$.

$$= \iint_D \sqrt{1 + (-2x)^2 + (-2y)^2} \, dA$$

where $D$.

$$= \iiint_D \sqrt{1 + \frac{(-2x)^2 + (-2y)^2}{4x^2 + 4y^2}} \, dA$$

where $D$. 

$$= \iiint_D \sqrt{1 + \frac{1}{4}} \, dr \, d\theta$$

SEE OTHER SIDE FOR MORE PROBLEMS
3. (6 points) Evaluate the iterated integral
\[
\int_0^2 \int_0^{\ln x} \int_0^{2z} xe^{-y} dy 
\]

\[
= \int_0^2 \int_0^{2z} x \left( \int_0^{\ln x} e^{-y} dy \right) dx dz = \int_0^2 \int_0^{2z} x \left( -e^{-y} \right) \bigg|_{y=0}^{y=\ln x} \ dx \ dz
\]

\[
= \int_0^2 \int_0^{2z} x e^{-\ln x} \ dx \ dz = \int_0^2 \int_0^{2z} \left( \frac{e^{-D}}{\ln x} - \frac{e^{-\ln x}}{\ln x} \right) \ dx \ dz
\]

\[
= \int_0^2 \int_0^{2z} x \left( 1 - \frac{1}{x} \right) \ dx \ dz = \int_0^2 \int_0^{2z} (x-1) \ dx \ dz
\]

\[
= \int_0^2 \int_0^{2z} \left( \frac{x^2}{2} - x \right) \ dx \ dz
\]

\[
= \int_0^2 \left[ \frac{x^2}{2} - x \right]_0^{2z} \ dz = \int_0^2 \left( \frac{4z^2}{2} - 2z \right) \ dz
\]

\[
= \left[ 2 \frac{z^3}{3} - z^2 \right]_0^2 = \frac{16}{3} - 4 = \frac{4}{3}
\]