(7 points) Identify the surface whose equation is given as $2r^2 + z^2 = 1$.

Solution: Since $2r^2 + z^2 = 1$ and $r^2 = x^2 + y^2$, we have $2(x^2 + y^2) + z^2 = 1$, an ellipsoid centered at the origin with intercepts $x = \pm \frac{1}{\sqrt{2}}$, $y = \pm \frac{1}{\sqrt{2}}$ and $z = \pm 1$.

(8 points) Evaluate

$$\iiint_{B} (x^2 + y^2 + z^2) \, dV,$$

where $B$ is the ball with center the origin and radius 5.

Solution: In spherical coordinates, $B$ is represented by $\{(\rho, \theta, \phi) | 0 \leq \rho \leq 5, 0 \leq \theta \leq 2\pi, 0 \leq \phi \leq \pi \}$. Thus

$$\iiint_{B} (x^2 + y^2 + z^2) \, dV = \int_{0}^{\pi} \int_{0}^{2\pi} \int_{0}^{5} (\rho^2) \rho^2 \sin \phi \, d\rho \, d\theta \, d\phi$$

$$= \int_{0}^{\pi} \sin \phi \, d\phi \times \int_{0}^{2\pi} \, d\theta \times \int_{0}^{5} \rho^4 \, d\rho$$

$$= \left[- \cos \phi \right]_{0}^{\pi} \times [\theta]_{0}^{2\pi} \times \left[\frac{1}{5} \rho^5 \right]_{0}^{5}$$

$$= (2)(2\pi)(625)$$

$$= 2500\pi$$