1. For $A$ a square matrix, let $\kappa_p(A)$ denote the condition number with respect to the matrix $l_p$ norm ($1 \leq p \leq \infty$). Show that

$$\kappa_1(A) \leq n\kappa_2(A), \quad \kappa_2(A) \leq n\kappa_1(A),$$

for all $A \in \mathbb{R}^{n \times n}$. Thus a matrix is well-conditioned or badly-conditioned with respect to the $l_1$ norm, if and only if it is with respect to the $l_2$ norm (up to a factor of the matrix size). State and prove a result of this type relating the condition numbers with respect to the $l_1$ norm and the $l_\infty$ norm.

2. In computing a cubic spline interpolant using equally spaced interpolation points, with spacing $h$, we are led to solving a linear system with tridiagonal matrix

$$A = \begin{pmatrix} 2h/3 & h/6 & & & \\ h/6 & 2h/3 & h/6 & & \\ & h/6 & 2h/3 & \cdots & \\ & & \cdots & \cdots & h/6 \\ & & & h/6 & 2h/3 \end{pmatrix} \in \mathbb{R}^{n \times n}.$$  

Prove that $\kappa_\infty(A) \leq 3$ (independent of the spacing $h$ or the number of points).

3. Write a column oriented algorithm to compute the Doolittle LU decomposition. That is, given a matrix $A$ with nonsingular principal minors, your algorithm should overwrite the elements on or above the diagonal of $A$ with the corresponding elements of an upper triangular matrix $U$ and overwrite the below-diagonal elements of $A$ with the corresponding elements of a unit lower triangular matrix $L$ such that $LU = A$, and your algorithm should access the elements of $A$ by columns. In addition to writing the algorithm, submit a direct Matlab translation along with a verification that it works using a random $4 \times 4$ matrix. Discuss the implementation of your algorithm using BLAS.