Periodic Table of the Finite Elements

The shape functions for \( P_r \) consist of all \( d \)-dimensional forms with \( r \) variables of degree \( r \), and the dimension of the space is \( \binom{d+r}{r} \).

This family is constructed from the complex of \( \mathbb{W}^{r} \)-conforming linear forms, using a nonconforming distance function \( d \cdot \mathbb{W}^{r} \).

The spaces with increasing degree \( r \) form a complex:

\[
P_{r} \subseteq P_{r+1} \subseteq \cdots
\]

The spaces with increasing degree \( r \) form a complex:

\[
S_{r} \subseteq S_{r+1} \subseteq \cdots
\]

The shape function space for \( S_{r} \) is given by

\[
S_{r}^C = \bigoplus_{d=1}^{n} \mathbb{W}^{r} \otimes S_{r-1}^{d} \otimes \cdots \otimes S_{r-1}^{1}
\]

where \( \mathbb{W}^{r} \) consists of homogeneous polynomials of degree \( r \).

\( S_{r}^{d} \) is the space of forms in \( \mathbb{W}^{r} \) that vanish on the boundary and conditioned on at least \( d \) points.

They are related to the \( S_{r} \) spaces by the isomorphism

\[
S_{r}^{d} \cong S_{r}^C \bigcap \mathbb{W}^{r-d}
\]

The spaces with increasing degree \( r \) form a complex:

\[
S_{r} \subseteq S_{r+1} \subseteq \cdots
\]