In the following questions, \( \# \) denotes the connected sum of surfaces.

1. Show that any open subset of a manifold is a manifold.

2. For each value of \( t \in \mathbb{R} \), decide whether the space
   \[
   \{(x, y, z) \in \mathbb{R}^3 \mid xyz = t\}
   \]
   is a manifold, and explain why or why not.

3. For which values of \( t \in \mathbb{R} \) is the space
   \[
   \{(x, y) \in \mathbb{R}^2 \mid x^2 + xy + ty^2 = 1\}
   \]
   a closed manifold?

4. If \( M \) is any surface and \( S^2 \) is the 2-sphere, explain why \( S^2 \# M \cong M \).

5. Show that, if \( P \) is a copy the real projective plane, \( P \# P \) is homeomorphic to a Klein bottle.