1. For maps $f : A \to B$ and $g : B \to C$ of abelian groups, show that there is an exact sequence

$$0 \to \ker(f) \to \ker(gf) \to \ker(g) \to \coker(f) \to \coker(gf) \to \coker(g) \to 0.$$ 

2. If $M$ is the Möbius strip and $\partial M$ is its boundary, calculate the relative homology groups $H_*(M, \partial M)$.

3. Show that there exists a natural transformation $\epsilon : C_0(X) \to \mathbb{Z}$, from the zero’th singular chain group of $X$ to $\mathbb{Z}$, satisfying $\epsilon \circ \partial = 0$, sending any point of $X$ to 1.

4. For a space $X$, we define the reduced singular chain complex of $X$ to be the chain complex

$$\cdots \to C_2(X) \to C_1(X) \to C_0(X) \xrightarrow{\epsilon} \mathbb{Z} \to 0$$

(i.e. defining $\widetilde{C}_{-1}(X) = \mathbb{Z}$), and the reduced homology groups $\tilde{H}_n(X)$ to be the homology groups of this complex. For most spaces, but not all, there is an isomorphism

$$H_0(X) \cong \mathbb{Z} \oplus \tilde{H}_0(X).$$

Figure out what the exceptional case is, and determine what the reduced homology groups are in this case.

5. A simplicial complex $(\mathcal{V}, \mathcal{F})$ is locally finite if, for all vertices $v \in \mathcal{V}$, there are only finitely many faces $\sigma \in \mathcal{F}$ containing $v$. In this circumstance, show that you can define groups $C_n^{BM}$ whose elements are arbitrary sums of $n$-simplices, together with boundary operators $\partial : C_n^{BM} \to C_{n-1}^{BM}$ satisfying $\partial \circ \partial = 0$. Calculate the associated homology groups $H_n^{BM}$ for a triangulation of $\mathbb{R}$. (These groups are called the Borel-Moore homology groups.)