1. Show graphically that the simplicial complex with 7 vertices, generated by the triangles below, gives rise to a space homeomorphic to the torus.

\[
\begin{array}{cccccccc}
123 & 127 & 134 & 145 & 156 & 167 & 236 \\
245 & 246 & 257 & 347 & 356 & 357 & 467 \\
\end{array}
\]

2. For a 2-dimensional simplicial complex with \(v\) vertices, \(e\) edges, and \(f\) triangles, the Euler characteristic \(\chi\) is defined to be \(v - e + f\). If this simplicial complex gives rise to a compact surface, give formulas for \(e\) and \(f\) in terms of \(\chi\) and \(v\) which are nondecreasing in \(v\).

3. Using the formulas from the previous problem, show that any triangulation of a compact surface of Euler characteristic 0 requires at least 7 vertices, and any of Euler characteristic 1 requires at least 6 vertices.

4. By identifying points on opposite sides of an icosahedron, give a triangulation of \(\mathbb{RP}^2\) with 6 vertices and 10 faces.

5. Suppose that you are given a simplicial complex with set \(V\) of vertices and set \(F \subset \mathcal{P}(V)\) of faces, satisfying two properties.

   (a) Any face \(U \in F\) satisfies \(|U| \leq 3\).

   (b) For any vertices \(a \neq b\) such that \(\{a, b\} \in F\), there are precisely two other vertices \(c\) such that the three-element set \(\{a, b, c\}\) is in \(F\).

Does this simplicial complex necessarily give rise to a compact surface? Either give a proof or a counterexample. (Be careful.)