1. Let \( f : M \to N \) be a map of smooth manifolds, and suppose \( N \) is equipped with a covariant vector field (given in local coordinates \( y^i \) by \( a_i dy^i \)). Show that this pulls back to a natural covariant vector field on \( M \).

2. Given a map \( f : M \to N \) of smooth manifolds and a vector field on \( M \) (given in local coordinates \( x^i \) by \( a^i \partial / \partial x^i \)), explain why this does not push forward to a natural vector field on \( N \). Give an example to illustrate this; if it is a complicated one, you are working too hard.

3. Let \( M \) be a smooth manifold. Given a smooth atlas of \( M \), give a smooth atlas on the tangent bundle \( TM \). Show that a smooth function \( f : M \to N \) gives rise to a smooth function \( df : TM \to TN \).

4. Suppose \( M \) is a smooth manifold and \( p : Y \to M \) is a covering map. Show that \( Y \) can be given the structure of a smooth manifold so that the projection map \( p \) is smooth.

5. Suppose that \( M \) is a smooth manifold and \( f \) is a smooth function from \( M \to \mathbb{R}^\text{v} \setminus \{0\} \). Give necessary and sufficient conditions for the smooth map

\[
p \mapsto f(p) / ||f(p)||
\]

from \( M \) to \( S^{n-1} \) to be

(a) a submersion,

(b) an immersion, or

(c) a local diffeomorphism.