1. Suppose $X$ is a connected, smooth manifold with basepoint $x$ and $Y$ is its universal cover, which inherits the structure of a smooth manifold together with an action of $\pi_1(X, x)$ by smooth maps. Show that, for any $n$, the projection map $\pi : Y \to X$ gives rise to an isomorphism

$$\pi^* : \Omega^n(X) \to [\Omega^n(Y)]^{\pi_1(X, x)}$$

between differential forms on $X$ and differential forms on $Y$ which are invariant under the action of $\pi_1(X, x)$.

2. Show that the wedge product $\wedge$ of differential forms induces a well-defined map $H_{dR}^p(M) \times H_{dR}^q(M) \to H_{dR}^{p+q}(M)$ which is associative and distributive over addition, making the de Rham cohomology into a ring.