1. Determine whether the series converges or diverges.

\[ \sum_{k=1}^{\infty} \frac{k \sin^2 k}{1 + k^3} \]

\[
\frac{k \sin^2 k}{1 + k^3} < \frac{k}{1 + k^3} < \frac{k}{k^3} = \frac{1}{k^2}
\]

Since \( \sum_{k=1}^{\infty} \frac{1}{k^2} \) converges (p-series with p=2>1), \( \sum_{k=1}^{\infty} \frac{k \sin^2 k}{1 + k^3} \) will also converge by the comparison test.

2. \[ \sum_{n=1}^{\infty} \frac{e^n}{n} \]

\[ e > 1 \text{ and } \lim_{n \to \infty} e^n = e^0 = 1, \text{ so } e^n > 1 \]

Thus \( \frac{e^n}{n} > \frac{1}{n} \)

Since \( \sum_{n=1}^{\infty} \frac{1}{n} \) is the harmonic series, which is divergent,

By the comparison test, \( \sum_{n=1}^{\infty} \frac{e^n}{n} \) is divergent.
3. \( \sum_{n=3}^{\infty} \frac{n+2}{(n+1)^3} \)

\[ \frac{n+2}{(n+1)^3} \approx \frac{1}{n^2} \] but we cannot easily see which is larger.

So we'll use the limit comparison test.

\[
\lim_{n \to \infty} \frac{n+2}{(n+1)^3} \cdot \frac{1}{n^2} = \lim_{n \to \infty} \frac{n^2(n+2)}{(n+1)^3} = \lim_{n \to \infty} \frac{n^3+2n^2}{n^3+3n^2+3n+1} \]

\[ = \lim_{n \to \infty} \frac{1 + \frac{2}{n}}{1 + \frac{3}{n} + \frac{3}{n^2} + \frac{1}{n^3}} = \frac{1+0}{1+0+0+0} = 1 > 0 \]

Thus, since \( \sum_{n=3}^{\infty} \frac{1}{n^2} \) is convergent (p-series \( p=2>1 \)),

by the limit comparison test,

\[ \sum_{n=3}^{\infty} \frac{n+2}{(n+1)^3} \] is also convergent.