1. Evaluate the following integrals.

   (a) \( \int \tan^7(x) \sec^3(x) \, dx \)  
   (b) \( \int_1^4 \frac{x^{3/2} \ln x \, dx}{x^3} \)  
   (c) \( \int \frac{dx}{x^3 + x} \)  
   (d) \( \int te^{\sqrt{t}} \, dt \)  
   (e) \( \int \frac{x^2 + 8x - 3}{x^3 + 3x^2} \, dx \)  
   (f) \( \int_0^{\pi/4} \frac{x \sin x \, dx}{\cos^3 x} \)

2. Evaluate the integral or show that it is divergent.

   (a) \( \int_0^3 \frac{dx}{x^2 - x - 2} \)  
   (b) \( \int_1^\infty \frac{\ln x \, dx}{x^4} \)  
   (c) \( \int_{-1}^1 \frac{1}{x^2 - 2x} \, dx \)

3. Find the exact length of the curve \( y = 2 \ln \left( \sin \left( \frac{x}{2} \right) \right), \pi/3 \leq x \leq \pi. \)

4. (a) The curve \( y = x^2, 0 \leq x \leq 1 \) is rotated about the \( y \)-axis. Find the area of the resulting surface.
   (b) Find the area of the surface obtained by rotating the curve in part (a) about the \( x \)-axis.

5. Section 9.2 problems 2 and 3-6

6. (a) Use Euler’s Method with step size 0.2 to estimate \( y(0.4) \), where \( y(x) \) is the solution to the initial value problem \( y' = 2xy^2, y(0) = 1. \)
   (b) Repeat part (a) with the step size 0.1.
   (c) Find the exact solution of the differential equation and compare the value at 0.4 with the approximations in parts (a) and (b).

7. Solve the differential equation or initial value problem.

   (a) \( (3y^2 + 2y)y' = x \cos x \)  
   (b) \( xy' = \ln x, y(1) = 2 \)

8. A bacteria culture starts with 1000 bacteria and the growth rate is proportional to the number of bacteria. After 2 hours, the population is 9000.

   (a) Find an expression for the number of bacteria after \( t \) hours.
   (b) Find the number of bacteria after 3 hours.
   (c) Find the rate of growth after 3 hours.
   (d) How long does it take for the number of bacteria to double?

9. Sketch the parametric curve \( x = 2 \cos \theta, \ y = 1 + \sin \theta \), and eliminate the parameter to get a Cartesian equation.

10. Sketch the polar curve.

    (a) \( r^2 = \sec 2\theta \)  
    (b) \( r = \frac{1}{1 + \cos \theta} \)

11. Find \( \frac{dx}{dy} \) and \( \frac{d^2y}{dx^2} \):  
    \( x = t \cos t, \ y = t \sin t. \)

12. Find the area enclosed by the curve \( r^2 = 9 \cos 5\theta. \)

13. Find the area enclosed by the inner loop of the curve \( r = 1 - 3 \sin \theta. \)

14. Find the length of the curve:  
    \( x = 3t^2, \ y = 2t^3, \ 0 \leq t \leq 2. \)
15. Find a formula for the sequence \{2, 7, 12, 17, \ldots\}.

16. Determine if the sequence \{\arctan(2n)\} converges or not.

17. Determine whether the series \(3 + 2 + \frac{4}{3} + \frac{8}{9} + \ldots\) is convergent or divergent.

18. Find the sum of the series \(\sum_{n=1}^{\infty} \frac{2^{2n+1}}{5^n}\)

19. Find the value(s) of \(x\) for which the series \(\sum_{n=0}^{\infty} 4^n x^n\) is convergent.

20. Determine whether each of the following series are absolutely convergent, conditionally convergent, or divergent:
   
   \[
   \begin{align*}
   (a) \quad &\sum_{n=1}^{\infty} ne^{-n} & (c) \quad &\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2 + 1} & (e) \quad &\sum_{n=1}^{\infty} (-1)^n \frac{n}{10^n} \\
   (b) \quad &\sum_{n=1}^{\infty} \frac{1}{n^3 + n} & (d) \quad &\sum_{n=1}^{\infty} \frac{n - 1}{n4^n} & (f) \quad &\sum_{n=1}^{\infty} \frac{n^n}{3^{1+3n}}
   \end{align*}
   \]

21. Determine the radius and interval of convergence for each of the following power series.

   \[
   \begin{align*}
   (a) \quad &\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n^3} & (b) \quad &\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}} & (c) \quad &\sum_{n=2}^{\infty} \frac{x^n}{(\ln n)^n}
   \end{align*}
   \]

22. Find a power series representation and the interval of convergence for each of the functions.

   \[
   \begin{align*}
   (a) \quad &f(x) = \frac{1}{x - 5} & (b) \quad &f(x) = \frac{3}{x^2 + x - 2} & (c) \quad &f(x) = \frac{x^2}{(1 - 2x)^2}
   \end{align*}
   \]

23. Find the Taylor series for the function centered at \(a\), and determine its interval of convergence.

   \[
   \begin{align*}
   (a) \quad &f(x) = \frac{1}{\sqrt{x}}, \quad a = 9 & (b) \quad &f(x) = \ln(1 - x), \quad a = 0
   \end{align*}
   \]

24. Which of the points \(P = (6, 2, 3), Q = (-5, -1, 4), \) and \(R = (0, 3, 8)\) is closest to the \(xy\)-plane? Which lies in the \(yz\)-plane?

25. Show that the equation \(x^2 + y^2 + z^2 = z + y + z\) represents a sphere. Find its center and radius.

26. Describe in words the region in \(\mathbb{R}^3\):

   \[
   \begin{align*}
   (a) \quad &y = -4 & (b) \quad &x^2 + y^2 + z^2 > 1 & (c) \quad &x^2 + y^2 + z^2 - 2z \leq 3
   \end{align*}
   \]

27. Find \( \overrightarrow{AB} \), then draw \( \overrightarrow{AB} \) and the equivalent vector from the origin.

   \[
   \begin{align*}
   (a) \quad &A = (2, 3), \quad B = (-2, 1) & (b) \quad &A = (-1, -1), \quad B = (-3, 4)
   \end{align*}
   \]
28. Find \(|a|, a + b, a - b, 2a, 3a + 4b,\) and the unit vector with the same direction as \(b:\)
   
   (a) \(a = \langle -4, 3 \rangle, \quad b = \langle 6, 2 \rangle\) 
   
   (b) \(a = \langle 6, 2, 3 \rangle, \quad b = \langle -1, 5, -2 \rangle\) 

29. Find the angle between the vectors.

   (a) \(\langle -8, 6 \rangle\) and \(\langle \sqrt{7}, 3 \rangle\) 
   
   (b) \(j + k\) and \(i + 2j - 3k\) 
   
   (c) \(\langle 1, 2, 3 \rangle\) and \(\langle 4, 0, -1 \rangle\) 

30. Find the scalar and vector projections of \(b = \langle 1, 1, 1 \rangle\) onto \(a = \langle 4, 2, 0 \rangle\).

31. Find \(a \times b\) and verify that it is orthogonal to both \(a = \langle 1, 2, 0 \rangle\) and \(b = \langle 0, 3, 1 \rangle\).

32. Find two unit vectors orthogonal to both \(\langle 1, -1, 1 \rangle\) and \(\langle 0, 4, 4 \rangle\).

33. Find the vector equation and parametric equations for the line through the point \((-2, 4, 10)\) and parallel to the vector \(\langle 3, 1, -8 \rangle\).

34. Find the parametric equations and symmetric equations of the line through the points \((1, 3, 2)\) and \((-4, 3, 0)\).

35. Find the vector, scalar and linear equations of the plane.

   (a) through the point \((6, 2, 3)\) and perpendicular to \(\langle -2, 1, 5 \rangle\).

   (b) through the point \((1, -1, 1)\) with normal vector \(i + j - k\).

   (c) through the point \((4, -2, 3)\) and parallel to the plane \(3x - 7z = 12\).

36. Find parametric equations for the line segment from \((10, 3, 1)\) to \((5, 6, -3)\).

37. Find the area of the parallelogram with vertices \(A = (1, 2, 3), \quad B = (1, 3, 6), \quad C = (3, 8, 6), \quad D = (3, 7, 3)\).

38. Determine whether the planes \(x + y + z = 1\) and \(x - y + z = 1\) are parallel, perpendicular, or neither. If neither, find the angle between them.

39. Find the limit. \(\lim_{t \to 0^+} \langle \cos t, \sin t, t \ln t \rangle\).

40. Sketch the curve with the given vector equation. Indicate with an arrow, the direction in which \(t\) increases.

   (a) \(r(t) = \langle t^4 + 1, t \rangle\) 

   (b) \(r(t) = ti + \cos 2tj + \sin 2tk\)

41. Find a vector equation and parametric equations for the line segment from \(P\) to \(Q\).

   (a) \(P = (0, 0, 0), \quad Q = (1, 2, 3)\) 

   (b) \(P = (1, -1, 2), \quad Q = (4, 1, 7)\)

42. Sketch the plane curve with the given vector equation. Find \(r'(t)\). Sketch the position vector \(r(t)\) and the tangent vector \(r'(t)\) for the given value of \(t\).

   (a) \(r(t) = \langle 1 + t, t^2 \rangle, \quad t = 1\) 

   (b) \(r(t) = e^t i + e^t j, \quad t = 0\)
43. Find the derivative of the vector function.
   (a) \( \mathbf{r}(t) = \mathbf{i} - \mathbf{j} + e^{4t} \mathbf{k} \)
   (b) \( \mathbf{r}(t) = (t^2, 1 - t, \sqrt{t}) \)

44. Find parametric equations for the tangent line to the curve \( x = t^5, y = t^4, z = t^3 \) at the point \( (1, 1, 1) \).

45. Evaluate the following integrals.
   (a) \( \int_{0}^{1} \left( 16t^3 \mathbf{i} - 9t^2 \mathbf{j} + 25t^4 \mathbf{k} \right) dt \)
   (b) \( \int \left( e^t \mathbf{i} + 2t \mathbf{j} + \ln(t) \mathbf{k} \right) dt \)

46. Find the length of the curve \( \mathbf{r}(t) = \langle 2t, t^2, \frac{1}{3}t^3 \rangle \), \( 0 \leq t \leq 1 \).

47. Given the vector function \( \mathbf{r}(t) = \langle 2 \sin t, 5t, 2 \cos t \rangle \).
   (a) Find the unit tangent and unit normal vectors \( \mathbf{T}(t) \) and \( \mathbf{N}(t) \).
   (b) Find the curvature.

48. Find the curvature of \( \mathbf{r}(t) = t^2 \mathbf{i} + t \mathbf{k} \).