MATH 1272 – Summer 2013

Name: ____________________________

Quiz 7 – Monday, July 22, 2013

You may not use a calculator.

1. (5 points) Determine whether the series $\sum_{n=1}^{\infty} \frac{\tan^{-1}(n)}{n^{3/2}}$ converges or diverges.

$0 < \tan^{-1}(n) \leq \frac{\pi}{2}$ for all $n$, so $\frac{\tan^{-1}(n)}{n^{3/2}} \leq \frac{\pi/2}{n^{3/2}} \approx \frac{1}{n^{3/2}} = \frac{1}{n^{3/2}}$

Now the limit comparison test can be used:

$$\lim_{n \to \infty} \frac{\tan^{-1}(n)}{n^{3/2}} = \lim_{n \to \infty} \frac{\tan^{-1}(n)}{n^{3/2}} \cdot \frac{n^{3/2}}{1} = \lim_{n \to \infty} \tan^{-1}(n) = \frac{\pi}{2} > 0$$

So, by the Limit Comparison Test, since $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ converges ($p$-series with $p = \frac{3}{2} > 1$),

then, $\sum_{n=1}^{\infty} \frac{\tan^{-1}(n)}{n^{3/2}}$ also converges.
2. (5 points) Decide whether the series \( \sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n} \) is absolutely convergent, conditionally convergent, or divergent. Justify your answer.

The root test applies:

\[
\lim_{n \to \infty} \sqrt[n]{\left| \frac{(-2)^{2n}}{n^n} \right|} = \lim_{n \to \infty} \sqrt[n]{\frac{(-2)^2}{n}} = \lim_{n \to \infty} \frac{4}{n} = 0 < 1.
\]

So, by the Root Test, the series \( \sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n} \) is absolutely convergent.