1. Bob eats lunch at the Coffman food court every week day. He either eats at Panda Express (P), Chick-fil-A (C), or Baja Sol (B). His transition matrix is

\[
\begin{array}{ccc}
P & C & B \\
\hline
P & 0.15 & 0.6 & 0.25 \\
C & 0.4 & 0.1 & 0.5 \\
B & 0.1 & 0.3 & 0.6 \\
\end{array}
\]

He ate Chinese food on Monday. What are the probabilities for his three meal choices of Friday (4 days later)?

2. Three of every four trucks on the road are followed by a car, while only one of every five cars is followed by a truck. What fraction of vehicles on the road are trucks?

3. To make a crude model of a forest we might introduce states 0 = grass, 1 = bushes, 2 = small trees, and 3 = large trees, and write down a transition matrix such as the following:

\[
\begin{array}{cccc}
0 & 1 & 2 & 3 \\
\hline
0 & 1/2 & 1/2 & 0 & 0 \\
1 & 1/24 & 7/8 & 1/12 & 0 \\
2 & 1/36 & 0 & 8/9 & 1/12 \\
3 & 1/8 & 0 & 0 & 7/8 \\
\end{array}
\]

The idea behind this matrix is that if left undisturbed, a grassy area will see bushes grow, then small trees, which of course grow into large trees. However, disturbances such as tree falls or fires can reset the system to state 0. Find the limiting fraction of land in each of the states.

4. When a basketball player makes a shot then he tries a harder shot the next time and hits (H) with probability 0.4 and misses (M) with probability 0.6. When he misses he is more conservative the next time and hits (H) with probability 0.7 and misses (M) with probability 0.3.

(a) Write the transition matrix for the Markov chain with state space \{H, M\}.
(b) Find the long-run fraction of time he hits a shot.

5. The Macrosoft Company gives each of its employees the title of programmer (P) or project manager (M). In any given year 70% of programmers remain in that position, 20% are promoted to project manager, and 10% are fired (state X). 95% of project managers remain in that position, while 5% are fired. How long on average does a programmer work before they are fired?
6. Suppose that \( \{X_n\} \) is a Markov chain with state space \( S = \{1, 2\} \), transition matrix
\[
P = \begin{pmatrix}
1/5 & 4/5 \\
2/5 & 3/5
\end{pmatrix},
\]
and initial distribution \( P(X_0 = 1) = 3/4 \) and \( P(X_0 = 2) = 1/4 \). Compute the following:
(a) \( P(X_3 = 1|X_1 = 2) \)
(b) \( P(X_3 = 1|X_2 = 1, X_1 = 1, X_0 = 2) \)
(c) \( P(X_2 = 2) \)
(d) \( P(X_0 = 1, X_2 = 1) \)

7. Let \( P \) be the transition probability matrix of a Markov chain. Argue that if for some positive integer \( r \), \( P^r \) has all positive entries, then so does \( P^n \) for all integers \( n \geq r \).

8. Specify the communicating classes of the following Markov chains and determine whether they are transient or recurrent:
\[
P_1 = \begin{pmatrix}
0 & 1/2 & 1/2 \\
1/2 & 0 & 1/2 \\
1/2 & 1/2 & 0
\end{pmatrix} \quad P_2 = \begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
1/2 & 1/2 & 0 & 0 \\
0 & 0 & 1 & 0
\end{pmatrix}
\]
\[
P_3 = \begin{pmatrix}
1/2 & 0 & 1/2 & 0 & 0 \\
1/4 & 1/2 & 1/4 & 0 & 0 \\
1/2 & 0 & 1/2 & 0 & 0 \\
0 & 0 & 0 & 1/2 & 1/2 \\
0 & 0 & 0 & 1/2 & 1/2
\end{pmatrix} \quad P_4 = \begin{pmatrix}
1/4 & 3/4 & 0 & 0 & 0 \\
1/2 & 1/2 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1/3 & 2/3 & 0 \\
1 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

9. A professor continually gives exams to her students. She can give three possible types of exams, and her class is graded as either having done well or poorly. Let \( g_i \) denote the probability that the class does well on a type \( i \) exam, and suppose that \( g_1 = 0.3, g_2 = 0.6, \) and \( g_3 = 0.9 \). If the class does well on an exam, then the next exam is equally likely to be any of the three types. If the class does poorly, then the next exam is always type 1. What proportion of exams are type \( i, i = 1, 2, 3 \)?

10. Consider a Markov chain \( \{X_n\} \) with states \( \{1, 2, 3\} \) and transition matrix
\[
P = \begin{pmatrix}
1/2 & 1/6 & 1/3 \\
1/4 & 1/2 & 1/4 \\
1/8 & 3/8 & 1/3
\end{pmatrix}
\]
Given that we start in state 1, what is the expected time it takes to reach state 3?