1. Let $V$ be the solution space of the linear system
\[
\begin{align*}
x_1 + 2x_2 + 3x_3 &= 0 \\
2x_1 + 2x_2 + 2x_3 &= 0
\end{align*}
\]
Find the dimension and a basis of $V$.
Answer: $\text{dim}(V) = 1$; vector $(1, -2, 1)$ can be taken as a basis.

2. Let $V$ be the solution space of the linear system
\[
\begin{align*}
x_1 + 2x_2 + 3x_3 + 4x_4 &= 0 \\
x_1 + x_2 + x_3 - x_4 &= 0 \\
x_2 + 2x_3 + 5x_4 &= 0
\end{align*}
\]
Find the dimension and a basis of $V$.
Answer: $\text{dim}(V) = 2$; vectors $(1, -2, 1, 0)$ and $(6, -5, 0, 1)$ form a basis.

3. Determine whether vectors $(0, 1, 1)$, $(2, 2, 0)$, $(1, 0, 3)$ are linearly dependent or not.
Answer: they are linearly independent.

4. Let $V$ be the subspace of $\mathbb{R}^3$ spanned by vectors $(1, 0, -1)$, $(1, 1, 1)$, $(0, 1, 2)$, $(2, 1, 0)$.
Find the dimension and a basis of $V$.
Answer: $\text{dim}(V) = 2$.

5. Show that any subspace of $\mathbb{R}^3$ containing vectors $(1, 2, 3)$, $(4, 5, 6)$ must contain vector $(7, 8, 9)$ as well.

6. Determine which of the following statements are true.
   a) any linearly independent collection of $n$ vectors forms a basis of $\mathbb{R}^n$;
   b) if vectors $\mathbf{u}, \mathbf{v}$ are linearly independent, then vectors $2\mathbf{u}, \mathbf{u} - \mathbf{v}$ are linearly independent as well;
   c) if vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are linearly dependent, then $\mathbf{w}$ can always be represented as a linear combination of $\mathbf{u}, \mathbf{v}$;
   d) two bases of the same space must have the same number of elements;
   e) if the number of variables in a system of linear equations is greater than the number of equations, then the system has a solution;
   f) if the number of variables in a system of linear equations is less than the number of equations, then the system does not have a solution;
   g) if the number of variables in a homogeneous system of linear equations is greater than the number of equations, then the system has a non-zero solution;
   h) if the number of variables in a homogeneous system of linear equations is less than the number of equations, then the system has a non-zero solution;
i) if $M$ is a 3-by-3 matrix such that $\det(M) \neq 0$ then one can always find a vector $v$ such that $v \cdot M = (1, 2, 3)$;

j) if $A$ and $B$ are invertible matrices of the same size, then $AB$ is also invertible;

k) if $A$ and $B$ are invertible matrices of the same size, then $A + B$ is also invertible;

l) if vectors $v_1, \ldots, v_n$ are linearly independent, then vectors $v_1, \ldots, v_{n-1}$ are linearly independent as well;

m) let $V$ be the space spanned by some vectors $v_1, \ldots, v_n$; then $\dim(V) \leq n$.

Answer: a) true; b) true; c) false; d) true; e) false; f) false; g) true; h) false; i) true; j) true; k) false; l) true; m) true.

7. Find the general solution of the differential equation $y'' + 4y' - 10y = 0$.

8. Find the solution of the differential equation $y'' - 2y' + y = 0$ subject to conditions $y(0) = 1, y'(0) = -2$. 

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