**Topic:** limit of a sequence.

**Definition.** A sequence \( \{x_n\} \) is said to have a number \( L \) as its limit if for any \( \epsilon > 0 \), there exists a positive integer \( n_0 \) such that \( |x_n - L| < \epsilon \) for all \( n \geq n_0 \).

**Remarks.**

1. The common notation for the limit of a sequence \( \{x_n\} \) is \( \lim_{n \to \infty} x_n \) or simply \( \lim x_n \).
2. Not all sequences have limits. Some simple examples of such "bad" sequences are 1, 0, 1, 0 \ldots and 1, 2, 3, 4 \ldots If a sequence \( \{x_n\} \) does not have a limit, then it is said to diverge. Otherwise, if \( \lim x_n \) exists, then we say that \( \{x_n\} \) converges.

The following facts are useful for finding limits of sequences:

- \( \lim(x_n \pm y_n) = \lim x_n \pm \lim y_n \)
- \( \lim x_n \cdot y_n = \lim x_n \cdot \lim y_n \)
- If \( y_n \neq 0 \) for any \( n \) and \( \lim y_n \neq 0 \), then \( \lim \frac{x_n}{y_n} = \frac{\lim x_n}{\lim y_n} \).
- If \( x_n \leq y_n \) for all \( n \), then \( \lim x_n \leq \lim y_n \).
- If \( f = f(x) \) is a continuous function, then \( \lim f(x_n) = f(\lim x_n) \).

1. For each of the following sequences, determine if it converges or diverges. If a sequence converges, find its limit.

   a) \( a_n = \frac{1}{n^2 - 10} \)

   b) \( b_n = \frac{n}{n^2 + n + 1} \)

   c) \( c_n = \frac{n^2 + 3}{5n^2 + 2n + 1} \)

   d) \( d_n = 0.99^n \)

   e) \( e_n = 1.01^n \)
f) \( f_n = \frac{5^n}{2^n + 3^n} \)

g) \( g_n = \cos \sqrt{2013} \)

h) \( h_n = \frac{n^3}{3^n} \)

2. Show that the sequence

\[
1, \frac{1}{2}, 1, \frac{1}{3}, 1, \frac{1}{4}, 1, \frac{1}{5} \ldots
\]

does not have a limit.

3. On January 1, 2013 John wrote number 100 on a blackboard. Every morning, starting the next day, he would erase the number \( x \) written on the board and write the number \( \sqrt{2x - 1} \) instead. What number (approximately) will be on the board on December 31, 2015?