Topic: limit of a sequence.

1. For each of the following sequences, determine if it converges or diverges. If a sequence converges, find its limit.
   a) \( a_n = \sqrt{n^2 + n} - n \)
   b) \( b_n = \frac{n \sin 2n - n^2}{n \cos 2n + n^2} \)
   c) \( c_n = \frac{2^n}{n^n} \)
   d) \( d_n = \frac{5 + 0.3^n}{3 + 0.5^n}; \) how about \( d'_n = \frac{5 + 0.3^n + n}{3 + 0.5^n - n} \)?
   e) \( e_n = \sin \ln(n + 1) - \sin \ln n \)

2. a) Give an example of sequences \( \{s_n\}, \{t_n\} \) such that \( \lim s_n = 0, \lim t_n = 0 \) and \( \lim \frac{s_n}{t_n} = 100 \).
   b) Give an example of sequences \( \{s_n\}, \{t_n\} \) such that \( \lim s_n = 0, \lim t_n = 0 \) and \( \lim \frac{s_n}{t_n} = 0 \).
   c) Give an example of sequences \( \{s_n\}, \{t_n\} \) such that \( \lim s_n = 0, \lim t_n = 0 \) and \( \lim \frac{s_n}{t_n} \) does not exist.
3. True or false?

a) If \( \{s_n\} \) and \( \{t_n\} \) are divergent sequences, then the sequence \( \{s_n + t_n\} \) is divergent as well.

b) If \( \{s_n\} \) and \( \{t_n\} \) are divergent sequences, then the sequence \( \{s_n \cdot t_n\} \) is divergent as well.

c) If \( \{s_n\} \) converges and \( \{t_n\} \) diverges, then the sequence \( \{s_n + t_n\} \) diverges.

d) If the sequence \( s_1, s_2, s_3, \ldots \) converges, then the sequence \( s_2, s_4, s_6, \ldots \) converges as well.

e) If \( \lim s_n t_n = 0 \), then \( \lim s_n = 0 \) or \( \lim t_n = 0 \).

4. A student lives 0.6 miles away from campus and 0.4 miles away from a pub. Once, at noon, he went to class\(^1\), but changed his mind after ten minutes of walking and decided to go to the bar instead. Five minutes later, he changed his mind back and decided to go to campus again. After walking for two and a half minutes, he turned back to the bar. He felt guilty a minute and 15 seconds later and decided to go to class again, and so on...

Where will the student be at 12:20pm if he walks at a speed of 3 miles per hour?

\(^1\)Sources say Calc II.