Worksheet 14

Topics: partial derivatives, tangent planes.

1. 
   a) Let $u(x, y) = \ln(e^x + e^y)$. Compute $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}$.

   b) Let $u(x, y, z) = (x - y)(y - z)(z - x)$. Compute $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.

   Answer: a) 1; b) 0.

2. Let $u(x, y, z) = x^y z$. Find all first-order partial derivatives of $u$. 
Consider a surface $S$ defined by the equation $z = f(x, y)$. An equation of the plane tangent to $S$ at the point $(x_0, y_0, f(x_0, y_0))$ has the form

$$z = f(x_0, y_0) + \frac{\partial f}{\partial x}
\bigg|_{(x_0, y_0)} \cdot (x - x_0) + \frac{\partial f}{\partial y}
\bigg|_{(x_0, y_0)} \cdot (y - y_0),$$

provided that $f = f(x, y)$ is differentiable at $x = x_0$, $y = y_0$.

3. a) Find an equation of the plane tangent to the surface $z = \frac{x^2}{2} - xy + 1$ at the point $(2, 1, 1)$.

b) Find an equation of the plane tangent to the surface $x + 2y - \ln z + 4 = 0$ at the point $(2, -3, 1)$.

3. Find all points on the surface $z = x^2y + 3xy + 6x + 1$ where the tangent plane is
   a) horizontal; b) parallel to the plane $-3x + 2y + z = 10$.