Worksheet 10

Topics: determinants

1. Compute the determinant.
\[
\begin{vmatrix}
\text{Alice} & 0 & \text{Bob} \\
\text{loves} & \text{hates} & 0 \\
0 & \text{Alice} & \text{Bob}
\end{vmatrix}
\]

2. Find the inverse of the matrix using the determinants.
\[
\begin{bmatrix}
2 & 0 & 1 \\
1 & 1 & 0 \\
-2 & 3 & 1
\end{bmatrix}
\]

3. Given points \(A(1, 0), B(2, 2), C(4, 5), D(3, 3)\), find the area of the parallelogram \(ABCD\).

4. Given points \(O(0, 0), A(-2, 3), B(4, 2), C(2, 1)\), find the area of the quadrilateral \(OABC\).
5. Show by example that \( \det(A + B) \) might not be equal to \( \det(A) + \det(B) \).

6. Let \( A \) be a matrix such that \( A^5 = 0 \). Can \( A \) be invertible?

7. Let \( A \) and \( B \) be such matrices that \( AB \) is not invertible. Deduce that at least one of the matrices \( A \) and \( B \) is not invertible.

8. 