Worksheet 10

Topics: determinants

1. Compute the determinant.

\[
\begin{vmatrix}
\text{Alice} & 0 & \text{Bob} \\
\text{loves} & \text{hates} & 0 \\
0 & \text{Alice} & \text{Bob}
\end{vmatrix}
\]

2. Find the inverse of the matrix
\[
\begin{bmatrix}
2 & 0 & 1 \\
1 & 1 & 0 \\
-2 & 3 & 1
\end{bmatrix}
\]
using the determinants.

3. Given points \(A(1, 0), B(2, 2), C(4, 5), D(3, 3)\), find the area of the parallelogram \(ABCD\).

4. Given points \(O(0, 0), A(-2, 3), B(4, 2), C(2, 1)\), find the area of the quadrilateral \(OABC\).
5. Show by example that $\det(A + B)$ might not be equal to $\det(A) + \det(B)$.

6. Let $A$ be a matrix such that $A^5 = \begin{bmatrix} 2013 & 2013 \\ 2243 & 2243 \end{bmatrix}$. Can $A$ be invertible?

7. Let $A$ and $B$ be such matrices that $A \cdot B$ is not invertible. Deduce that at least one of the matrices $A$ and $B$ is not invertible.

8. Show that a matrix with two equal rows (or columns) has zero determinant.

9. Show that a matrix with two equal rows (or columns) has zero determinant.