Worksheet 14

Topics: dimension and bases.

1. Verify that the set \{(2, 0, 3), (1, -1, 2), (0, 1, 1)\} forms a basis of \(\mathbb{R}^3\). Find the coordinates of the vector (3, 3, 3) in this basis.

2. Let \(P_3\) be the space of polynomials of degree not greater than 3. Show that polynomials

\[1, 1 + x, 1 + x + x^2, 1 + x^2 + x^3\]

form a basis of this space. What are the coordinates of \(p(x) = 10 + x - x^3\) in this basis?

3. For each of the given spaces, find its dimension.

a) The subspace of \(\mathbb{R}^3\) consisting of vectors of the form \((x, y, 0)\).

b) The space of 3-by-3 upper-triangular matrices.

c) The space of polynomials \(p\) of degree not greater than 10 and such that \(p(20) = 0\).
4. Find the dimension and a basis the solution space of the given linear system.

a) \[
\begin{aligned}
    x_1 + 3x_2 + 4x_3 &= 0 \\
    3x_1 + 8x_2 + 7x_3 &= 0
\end{aligned}
\]

b) \[
\begin{aligned}
    x_1 + 3x_2 + 4x_3 &= 0 \\
    3x_1 + 8x_2 + 7x_3 &= 0 \\
    2x_1 + x_2 &= 0
\end{aligned}
\]

c) \[
\begin{aligned}
    x_1 + 3x_2 + 4x_3 &= 0 \\
    3x_1 + 8x_2 + 7x_3 &= 0 \\
    x_1 + 2x_2 - x_3 &= 0
\end{aligned}
\]

d) \[
\begin{aligned}
    x_1 - 3x_2 - 10x_3 + 5x_4 &= 0 \\
    x_1 + 4x_2 + 11x_3 - 2x_4 &= 0 \\
    x_1 + 3x_2 + 8x_3 - x_4 &= 0
\end{aligned}
\]