Worksheet 22

Topics: non-homogeneous LDEs; the method of variation of parameters.

How to find a particular solution of a non-homogeneous LDE $y'' + p(x)y' + q(x)y = f(x)$:

Step 1. Find the general solution of the associated homogeneous equation $y'' + p(x)y' + q(x)y = 0$. Say, $y_c = C_1y_1 + C_2y_2$.

Step 2. Set up the system of equations

\[
\begin{align*}
&u_1' y_1 + u_2' y_2 = 0 \\
u_1' y_1' + u_2' y_2' = f(x)
\end{align*}
\]

and solve it for $u_1'$, $u_2'$.

Step 3. Find $u_1$ and $u_2$ by taking integrals of $u_1'$ and $u_2'$ respectively. The function $y_p = u_1 y_1 + u_2 y_2$ will be a particular solution of the given LDE.

1. Find the general solutions of the given equations.

a) $y'' - 2y = 8e^x$

b) $y'' + 4y = \frac{1}{\sin 2x}$
c) \[ y'' - 2y' + y = \frac{e^x}{x} \]

d) \[ y'' + 2y' + 2y = \frac{e^{-x}}{\sin x} \]

e) \[ y'' + 3y' + 2y = \frac{1}{e^{x}+1} \]