Worksheet 20

Topics: systems of differential equations.

1. For the given systems of differential equations, sketch their direction fields and solution curves.
   a) \[ \begin{cases} x' = x + y \\ y' = 2y \end{cases} \]
   b) \[ \begin{cases} x' = 1 - y \\ y' = 1 - x \end{cases} \]

2. Transform a differential equation \( y''' + \sin y'' \cdot t + y'y = 0 \) into an equivalent system of first-order differential equations.

3. Find the general solution of the system \( \begin{cases} x' = 2y \\ y' = x - y \end{cases} \) by first transforming it into a single second-order differential equation.
4. Rewrite the systems in the matrix form $x'(t) = P x(t) + f(t)$.

a) $\begin{cases} x' = x + y \\ y' = x - y \end{cases}$
b) $\begin{cases} x' = -y + e^t \\ y' = x - \sin t \end{cases}$
c) $\begin{cases} x' = y + z + t \\ y' = x + z + t^2 \\ z' = x + y + t^3 \end{cases}$

5. Verify that $x_1 = \begin{bmatrix} e^t \\ -e^t \end{bmatrix}$ and $x_2 = \begin{bmatrix} e^{2t} \\ -2e^{2t} \end{bmatrix}$ are linearly independent solutions of the SDE $x' = \begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix} x$. Then find the solution of this SDE satisfying the initial condition $x(0) = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$. 