When we wish to reduce substances to their principles, we state that lines come from the short and long, and the plane from the broad and narrow, and body from the deep and shallow.

Aristotle

*Metaphysics*

**Worksheet 2**

The vector equation of the line \( l \) passing through the point \( A(x_0, y_0, z_0) \) in the direction of the vector \( \mathbf{v} = (a, b, c) \) is \( l(t) = A + tv \), where the parameter \( t \) takes on all real values. In coordinate form, the equation is \( (x, y, z) = (x_0, y_0, z_0) + t(a, b, c) \) or

\[
\begin{align*}
  x &= x_0 + at \\
  y &= y_0 + bt \\
  z &= z_0 + ct
\end{align*}
\]

1. Write an equation of the line \( l \).

   a) The line \( l \) passes through the points \( A(-1, 2, 3), B(2, 6, -2) \).

   b) The line \( l \) passes through the point \( A(0, 1, -3) \) parallel to the line \( m \) determined by the equations

   \[
   \begin{align*}
   x &= 3 + t \\
   y &= 1 - t \\
   z &= 2 + 3t
   \end{align*}
   \]

   c) The line \( l \) passes through the point \( A(1, 1, 2) \) perpendicular to the \( y \)-axis.

\[
\cdot (\mathbf{e} - \mathbf{e}' \cdot 1 - 1) = (\mathbf{e}' \cdot x) (\mathbf{e} \cdot (x + z - 1 - 1) = (\mathbf{e}' \cdot x) (\mathbf{e} \cdot (y - 2) \mathbf{e} + z \mathbf{e} + 1 -) = (\mathbf{e}' \cdot x) (\text{transpose})
\]
The equation of the plane $\alpha$ passing through the point $A(x_0, y_0, z_0)$ perpendicular to the vector $\mathbf{n} = (A, B, C)$ is

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0.$$ 

Vector $\mathbf{n}$ is called a normal vector.

2. Write an equation of the plane $\alpha$.
   
a) The plane $\alpha$ passes through the point $A(1, 0, -2)$ perpendicular to the line $l(t) = (2t, 1 - t, 3t)$.

   b) The plane $\alpha$ passes through the point $A(0, 4, 1)$ parallel to the plane $x - y + z = 100$.

   c) Find an equation of the plane containing the lines $c_1(t) = (1 + 2t, 2 - 3t, 3)$ and $c_2(t) = (-t, t + 3, 1 - 2t)$.

2. Determine if points $A(2, -1, -2)$, $B(1, 2, 1)$, $C(2, 3, 0)$, $D(5, 0, -6)$ lie in the same plane.