I guess that’s what happens in the end, you start thinking about the beginning.

John Smith in
Mr. & Mrs. Smith

Worksheet 15

How to find the extreme points of a function $f = f(x_1, x_2, \ldots, x_n)$ subject to a constraint $g(x_1, x_2, \ldots, x_n) = c$:

Step 1. Introduce a new variable $\lambda$ and define the auxiliary function $h$ by letting

$$h(x_1, x_2, \ldots, x_n, \lambda) = f(x_1, x_2, \ldots, x_n) - \lambda[g(x_1, x_2, \ldots, x_n) - c].$$

Step 2. Find and classify the extreme points of $h$ as in the unconstrained case.

This is the so-called Lagrange multipliers method.

How to find the extreme points of a function $f = f(x_1, x_2, \ldots, x_n)$ within the domain determined by an inequality $g(x_1, x_2, \ldots, x_n) \leq c$:

Step 1. Find the extreme points of $f$ as in the unconstrained case and exclude those which do not satisfy the given inequality.

Step 2. Now we need to check the points on the boundary. That means, find the extreme points of $f$ subject to the constraint $g(x_1, x_2, \ldots, x_n) = c$ (as discussed above).

1. Find the extrema of $f(x, y) = x^2 + y^2$ subject to the condition a) $\frac{x}{2} + \frac{y}{3} = 1$; b) $2y - x^2 \geq 0$.

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1 Joseph Louis Lagrange (1736-1813) was a French mathematician and astronomer of Italian descent. He made significant contributions to the theory of differential equations, calculus of variations, number theory, celestial mechanics.
2. Show that the largest rectangle that can be inscribed in a circle is a square.

3. What is the volume of the largest rectangular parallelepiped that can be inscribed in a hemisphere of radius $R$?

4. Find the shortest distance between the ellipse $x^2 + 4y^2 = 4$ and the line $2x + 3y - 6 = 0$. 