3.3 5

(a) 
\[
\begin{bmatrix}
    1 & 0 \\
    0 & 1 \\
\end{bmatrix}
+ \begin{bmatrix}
    1 & 0 \\
    0 & 1 \\
\end{bmatrix}
+ \begin{bmatrix}
    0 & 1 \\
    1 & 0 \\
\end{bmatrix}
+ \begin{bmatrix}
    0 & 1 \\
    0 & 1 \\
\end{bmatrix}
= \begin{bmatrix}
    0 & 0 \\
    0 & 0 \\
\end{bmatrix}
\]

\[ \implies C_1 + C_2 = 0 \]
It is easy to observe that \( C_1 = C_4 = 1 \) \( C_2 = C_3 = -1 \) are nontrivial solutions. Thus, it is linearly dependent.

(b) 
\[
\begin{bmatrix}
    1 & 0 \\
    0 & 1 \\
\end{bmatrix}
+ \begin{bmatrix}
    1 & 0 \\
    0 & 1 \\
\end{bmatrix}
+ \begin{bmatrix}
    0 & 1 \\
    1 & 0 \\
\end{bmatrix}
+ \begin{bmatrix}
    2 & -2 \\
    -1 & 1 \\
\end{bmatrix}
= \begin{bmatrix}
    0 & 0 \\
    0 & 0 \\
\end{bmatrix}
\]

\[ \implies \begin{cases}
    C_1 + C_2 + 2C_4 = 0 \\
    C_3 - 2C_4 = 0 \\
    C_1 + C_3 - C_4 = 0 \\
    C_2 + C_4 = 0
\end{cases} \implies C_1 + C_2 + C_3 = 0 \]
\[ \implies \text{There are arbitrary many solutions. } \implies \text{linearly dependent.} \]

7.

(a) 
Notice that \( \frac{d^2 y}{dt^2} - y = 0 \) is linear. And the axioms to justify a vector space implicate that if \( y_1 \) and \( y_2 \) are in \( V \), then linear combination \( ay_1 + by_2 \) is again in \( V \) for any choice of constants \( a \) and \( b \). So, \( V \) is a vector space in this problem clearly.
(b) 
\[ \frac{d^2 y}{dt^2} - y = 0 \]
The characteristic equation is \[ r^2 - 1 = 0 \Rightarrow r = \pm 1 \]
Thus, the two solutions are \[ y_1 = e^t, \quad y_2 = e^{-t} \]
Clearly, \( y_1 \) and \( y_2 \) are linearly independent. So \( y_1, y_2 \) form the basis.

10. Let's check \( x'(t), x^2(t), x^3(t) \) are solutions of the ODE, which is in \( V \). This is clear.
Then we need to show they are linearly independent.
Notice that \( e^t, e^{2t}, e^{3t} \) are linearly independent, then \( x'(t), x^2(t), x^3(t) \) are also linearly independent clearly. So they form a basis for \( V \).

3.6 10.
\[
A = \begin{pmatrix}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{pmatrix}
\]
\[ \det A = \begin{vmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{vmatrix} = 1 \neq 0 \]
The inverse of \( A \) exists.
\[
A^{-1} = \frac{\text{adj} A}{\det A} = \text{adj} A = \begin{pmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{pmatrix}
\]