Midterm for MFM 5022  Spring 2017

One page of notes (front and back) allowed. Simple calculator may be used.
1. The following questions require one line answers.

(a) True or False? Option $\Theta$ can be hedged using a short index position.

(b) True or False? Dollar deltas, $\Delta_d$ add across underlyings.

(c) True or False? The magnitude of Value at Risk is greater than Conditional Value at Risk for the same confidence level.

(d) True or False? In the presence of skew, the Black-Scholes $\Delta$ underestimates the true option $\Delta$.

(e) True or False? Implied probability of default is inversely related the price of bonds.

(f) True or False? Monte Carlo pricing techniques are best suited for American options.

(g) True or False? One advantage of a finite difference scheme for option pricing is that prices are available for multiple values of the underlying.

(h) True or False? The Cox-Ross-Rubinstein binomial tree showed that $u$ and $d$ are uniquely determined as $u = 1/d$ otherwise there are arbitrage opportunities.

(i) True or False? GARCH models are particularly useful since squared returns appear to cluster.

(j) Which numerical method would you use to price a European option that is path dependent?
2. If \( r \sim N(\mu, \Sigma) \), for \( \Pi = \mu \in \mathbb{R}^N \) and \( \Sigma \in \mathbb{R}^{N \times N} \), how is \( P_0 + \Delta_d' r \) distributed for constants \( P_0 \in \mathbb{R} \) and \( \Delta_d \in \mathbb{R}^N \)?

(a) Suppose the total dollar delta for options on separate underlyings is given by

\[
\Delta_d = \begin{pmatrix} 1500 \\ 2000 \end{pmatrix}
\]

with annualized covariance of underlying returns

\[
\Sigma = \begin{pmatrix} .090 & .048 \\ .048 & .040 \end{pmatrix}
\]

Using the table below, approximate the 5 day 95% VaR for these option exposures.

<table>
<thead>
<tr>
<th>( N^{-1}(0.05) )</th>
<th>-2.5758</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N^{-1}(0.01) )</td>
<td>-2.3268</td>
</tr>
<tr>
<td>( N^{-1}(0.025) )</td>
<td>-1.9600</td>
</tr>
<tr>
<td>( N^{-1}(0.05) )</td>
<td>-1.6499</td>
</tr>
</tbody>
</table>
3. Assume the risk free rate is 0.25% and recovery rates are 40% for a given set of bonds, and that a one year CDS protection on these bonds has a spread of 400 bps made once at the end of the year.

   (a) Write down the formula for the valuation of the fee leg (assuming no accruals) in terms of the probability of default, $p$.

   (b) Write down the formula for the valuation of the contingent leg in terms of the probability of default, $p$.

   (c) Relate the fee leg and contingent leg to derive an approximate relationship between spreads, recovery, and probability of default, similar (but not identical) to that derived in class. What is the one year probability of default in this case?
4. Suppose two securities share a common source of uncertainty and follow

\[
\frac{df_1}{f_1} = \mu_1 dt + \sigma_1 dz \\
\frac{df_2}{f_2} = \mu_2 dt + \sigma_2 dz,
\]

so that a small change in either security is given by

\[\Delta f_i = \mu_i f_i \Delta t + \sigma_i f_i \Delta z.\]

(a) Suppose you have a portfolio, \(\Pi\) long \((\sigma_2 f_2)\) units of \(f_1\) and short \((\sigma_1 f_1)\) units of \(f_2\). Write \(\Delta \Pi\) as \(\mu_{\Pi} \Delta t + \sigma_{\Pi} \Delta z\) for appropriate \(\mu_{\Pi}\) and \(\sigma_{\Pi}\).

(b) How is \(\Delta \Pi\) related to \(r\Pi \Delta t\)? Justify your answer.

5. Suppose one month European calls with strike prices \$5.00, \$5.25, and \$5.50 are \$3.00, \$2.92, and \$2.85, respectively. If the risk free rate is 8%, estimate the probability that the underlying will be between \$5.00 and \$5.50 in one month.
6. The GARCH(1,1) model is given by

\[ \sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2, \]

or

\[ \sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2. \]

Suppose you have used daily data and estimated that \( \omega = .0000003, \alpha = 0.18, \) and \( \beta = 0.80. \)

(a) What is \( \gamma? \)

(b) What is the long-run average annualized volatility implied by the model?

7. A portfolio of options, \( \Pi \) on a single underlying, \( S, \) satisfies the following differential equation

\[ \frac{\partial \Pi}{\partial t} + r S \frac{\partial \Pi}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 \Pi}{\partial S^2} = r \Pi. \]

If the portfolio is delta-neutral and has very large and positive gamma exposure, show that the theta of the portfolio is large and negative.

8. What is the empirical average five year hazard rate for bonds if the observed cumulative default probability is 10%
Use the rest for scratch