Midterm for MFM 5022  Spring 2018

One page of notes (front and back) allowed. Simple calculator may be used.
1. (a) True or False. CDS basis dynamics are largely driven by technicals – such as liquidity – rather than fundamentals.

(b) True or False. When determining the $\beta$-VaR of a portfolio with a jointly normal distribution of market invariants, we may often omit the mean.

(c) True or False. Antithetic variable techniques are especially effective when payoff functions are even.

(d) True or False. Finite difference schemes are preferred for path dependent options.

(e) True or False. The credit spread puzzle implies that bond prices are higher than historical probabilities of default indicate.

(f) True or False. Based on the average CDS hazard rate models shown in class, the term structure of hazard rates must be monotonically increasing otherwise there is a model arbitrage.

(g) True or False. The credit triangle holds exactly when accruals are ignored and hazard and interest rate discounting is continuous.

(h) True or False. Implied probability densities from option prices are based on the log-normal assumption of Black-Scholes.

(i) True or False. Option $\Gamma$ may be hedged by holding positions in other options on the same underlying.

(j) True or False. Binomial trees are well suited for American option pricing.
2. Suppose an index with constituent weights, \( w \in \mathbb{R}^N \), has volatility \( \sigma_0 \). Let the covariance of index members be given by

\[
\Sigma = \begin{pmatrix}
\sigma_1^2 & \ldots & \rho \sigma_1 \sigma_N \\
\vdots & \ddots & \vdots \\
\rho \sigma_N \sigma_1 & \ldots & \sigma_N^2
\end{pmatrix}.
\]

(a) Relate \( \sigma_0 \) to \( w \) and \( \Sigma \).

(b) Solve for \( \rho \) in terms of the constituent volatilities, \( \sigma_i \), and the index volatility, \( \sigma_0 \). (This is the so-called *implied correlation* and is traded in the market.)

3. Consider a portfolio of options, and assume there is a stock with index \( \beta \) of 1.50 that can’t be shorted with 10,000 options written on it and a \( \Delta \) of 0.75. The current stock price is $5. How many shares would you need to short of the index if the current index price is $225 to be \( \Delta \) neutral?
4. You have a portfolio of 20 options on a single underlying with a $\Delta$ of 0.60, $\Gamma$ of 0.25 and $\Theta$ of -3.

(a) If the underlying closed at $10$ two days ago and opened at $10.25$ today, approximately how much did the value of the portfolio change? (Assume there are 252 trading days a year.)

(b) Suppose you were short 12 shares of stock. How much would you estimate the portfolio moved?

5. Suppose 4 year bonds have a spread over treasuries of 820 bps. Using an empirical recovery value of 25% and the ad hoc recovery value of 40%, give a range of hazard rate approximations using the credit triangle approximation.
6. Suppose you have a portfolio of two stocks. There are 100 shares of Stock One and 200 shares of Stock Two, and they trade at $10 and $12, respectively. The annualized covariance of underlying returns is

\[
\Sigma = \begin{pmatrix}
0.16 & -0.96 \\
-0.96 & 0.09 \\
\end{pmatrix}
\]

Using the table below, approximate the 3 day 95% VaR for these option exposures.

<table>
<thead>
<tr>
<th>N \text{−} 1 \times 0.95</th>
<th>2.5758</th>
</tr>
</thead>
<tbody>
<tr>
<td>N \text{−} 1 \times 0.99</td>
<td>2.3268</td>
</tr>
<tr>
<td>N \text{−} 1 \times 0.975</td>
<td>1.9600</td>
</tr>
<tr>
<td>N \text{−} 1 \times 0.95</td>
<td>1.6499</td>
</tr>
</tbody>
</table>

7. If the spread on a five year CDS was put on as a seller at 300 bps and two years later has moved to 325 bps, what is the mark to market on $100,000 notional if the Risky PV01 is 2.20?

8. Using the piecewise average hazard rate models developed in class, if the average hazard rate from years 0 to 1 is \( \lambda_{0,1} = 0.05 \), and the average hazard rate from years 1 to 3 is \( \lambda_{1,3} = 0.07 \), what is the cumulative probability of survival at 2 years?
9. Motivated by GBM with no drift term, we know that $S_T = S_0 \exp \left( \frac{\sigma^2 T}{2} + \sigma \sqrt{T} Z \right)$, with $Z \sim N(0, 1)$.

(a) Find the two day 99% VaR for a portfolio with 50 shares of stock following GBM with an annual volatility of 30% and current price of $1$. (Hint: find $\Delta S$ and scale appropriately.)

(b) A Taylor expansion of $e^x$ is given by $x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots$. Expand $S_T$ up to the $\sqrt{T}$ term (i.e., omit all higher order terms). How does this expansion relate to VaR developed in class?

10. Suppose one year European calls with strike prices $2.00$, $2.50$, and $3.00$ are $1.90$, $1.81$, and $1.75$, respectively. If the risk free rate is 5%, estimate the probability that the underlying will be between $2.00$ and $2.50$ in one year.
11. Below, use the Cox-Ross-Rubinstein assumptions to fill in portions of a binomial tree for a binary option that pays one dollar if the underlying non-dividend paying stock is above $40 and nothing otherwise. Assume the risk free rate is 5% and the annualized volatility is 25% and the time step in the tree is ten days. (Use a 252 day count convention.)

(a) What are \( a, p, u, \) and \( d \)?

(b) Assume the right-most column of the table below is the terminal nodes of the tree. Determine the final payoffs under up and down moves, \( O_u \) and \( O_d \), respectively.

<table>
<thead>
<tr>
<th></th>
<th>40.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S = ? )</td>
<td>( O_u = ? )</td>
</tr>
<tr>
<td>( O_E = ? ), ( O_A = ? )</td>
<td>37.11</td>
</tr>
<tr>
<td></td>
<td>( O_d = ? )</td>
</tr>
</tbody>
</table>

(c) What is \( S \)? What are the values of the option, \( O_E \) and \( O_A \), at the node with \( S \) assuming European and American optionality, respectively?
Use the rest for scratch