1. Prove that when the \textit{ad hoc} assumption \( p = 0.5 \) is added to the system of equations

\[
\exp((r - q)\Delta t) = p \cdot u + (1 - p) \cdot d \\
\exp((r - q)\Delta t) (u + d) - u \cdot d = \exp(2(r - q)\Delta t) = \sigma^2 \Delta t,
\]

the resulting up and down parameters (up to first order in \( \Delta t \)) satisfying the system of equations are given by

\[
u = \exp \left( \left( r - q - \frac{\sigma^2}{2} \right) \Delta t + \sigma \sqrt{\Delta t} \right)
\]

\[
d = \exp \left( \left( r - q - \frac{\sigma^2}{2} \right) \Delta t - \sigma \sqrt{\Delta t} \right).
\]

Hint: Use the ansatz

\[
u = \exp \left( \alpha \Delta t + \beta \sqrt{\Delta t} \right)
\]

\[
d = \exp \left( \alpha \Delta t - \beta \sqrt{\Delta t} \right)
\]

and solve for \( \alpha \) and \( \beta \) by solving the system of equations above up to first order in \( \Delta t \).

2. (motivated by Hull 21.25) An American put option to sell the Swiss franc for dollars has a strike of $0.95, a time to maturity of six months, and volatility of the franc of 10%. The dollar interest rate is 6% and the Swiss franc interest rate is 4%, while the current exchange rate is 0.85.

(a) Use a 12 step binomial tree (using the standard CRR assumptions) to value the option.

(b) Estimate \( \Delta, \Gamma, \) and \( \Theta \).

(c) Estimate \( \nu \).

(d) Use a 12 step binomial tree and the \textit{ad hoc} tree model above to value the option.

3. (motivated by Hull 21.27) Price a six month American call option struck at $30 on a stock whose price is currently $28 which is expected to pay dividends of $1 on the second and fifth month. Assume a volatility of 15% annualized, use twelve time steps, and follow the methodology wherein stock prices are assumed to have two components: a part uncertain and a part that is the present value of dividends paid during the life of the option.

(a) Price the same option, but include another dividend payment at seven months and include this dividend in your present value determination.
4. These problems revisit the implied volatility data for one year options on an underlying with spot price 226.28 and risk free rate 0.35% given in

vol_skew.csv

(a) Plot the implied distribution using uniform distance in strike of 0.01 by using a quadratic interpolation.

(b) What is the implied probability that the price will be between 147.08 and 294.16?

(c) What is the probability that the price will be below 147.08?