1. Hull 29.21

2. Hull 29.22

3. Let the market price of risk be \( \lambda_0 \), let

\[
dr = m(r,t)dt + s(r,t)dZ,
\]

and let \( P(t,T,r) \) denote a zero coupon bond depending on \( r \) and \( t \) for maturity \( T \).

(a) Find \( \mu(t,T,r) \) and \( \sigma(t,T,r) \) if \( dP = \mu(t,T,r)Pdt + \sigma(t,T,r)PdZ \).

(b) Consider two different maturities, \( T_1 \) and \( T_2 \) and zero coupon bonds, \( P(t,T_1,r) \) and \( P(t,T_2,r) \). Let \( \mu_i = \mu(t,T_i,r) \) for \( i = 1, 2 \), and similarly for \( \sigma_i \) and \( P_i \). Construct a portfolio, \( \Pi \), that is long \( \sigma_2 P_2 \) units of the the bond maturing at \( T_1 \) and short \( \sigma_1 P_1 \) units of the bond maturing at \( T_2 \).

i. Show that

\[
d\Pi = (\sigma_2 P_2 \mu_1 P_1 - \sigma_1 P_1 \mu_2 P_2) dt
\]

ii. Why is \( d\Pi = r\Pi dt \)?

iii. Prove that \( \mu(t,T,r) = r + \lambda_0 \sigma(t,T,r) \) for all \( T \).

iv. Using the formulae for \( \mu(t,T,r) \) and \( \sigma(t,T,r) \) for \( dP \) in the first part of this question, and the preceding question, derive the PDE for bond prices:

\[
\frac{\partial P}{\partial t} + \frac{1}{2} \sigma^2 \frac{\partial^2 P}{\partial r^2} + (m - \lambda_0 s) \frac{\partial P}{\partial r} - rP = 0.
\]