The following problems will refer to the table of treasury bonds

<table>
<thead>
<tr>
<th>Maturity (years)</th>
<th>Yield to Maturity</th>
<th>Coupon (annual)</th>
<th>Current Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.5%</td>
<td>3.50</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>4.0%</td>
<td>4.00</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>4.5%</td>
<td>4.50</td>
<td>100</td>
</tr>
</tbody>
</table>

1. What are the annual spot rates determined by the reference treasury bonds?

2. What are the annual forward rates determined by the reference treasury bonds?

3. In class we outlined a procedure to build a rate tree based on dynamics

\[ d \ln r = \mu(\ln r, t) dt + \sigma dz \]

and showed that, for example, \( r_u = r_d e^{2\sigma \sqrt{\Delta t}} \).

(a) Calibrate a tree to the two year treasury bond, assuming \( \sigma = 0.08 \).
Verify that your tree prices the two year treasury bond with current market prices.

(b) Calibrate a tree to the two year treasury bond, matching both forwards in expectation as well as prices. What is \( \sigma_1 \)?

(c) Extend this tree to the three year treasury bond, matching its forwards and price. Verify that the tree prices each calibrated instrument according to the market. What is \( \sigma_2 \)?

(d) The following problems fix the tree calibrated in the preceding question.

i. Price a three year bond paying par in three years with an annual 5.25 coupon.
   A. If the market price is $101.10, what is the bond’s OAS?

ii. Price a three year callable bond, callable in years one and two for $99.50, with annual 5.25 coupons.
   A. If the market price of this callable bond is $99.75, what is its OAS? What is the effective duration? The effective convexity? What is the Yield to Call for this bond?

iii. Price a three year putable bond, putable in years one and two for $99.50, with annual 5.25 coupons and the same OAS as callable bond trading at $99.75.