1. True or False.
   
   (a) True or False. Equilibrium models of the short rate are designed to be calibrated to the forward rate curve.
   
   (b) True or False. The Ho-Lee model of the short rate produces a non-flat term structure of forward bond price volatility.
   
   (c) True or False. The duration of a portfolio of bonds is the weighted sum of the duration of the bonds.
   
   (d) True or False. Interest rate caps and floors may be valued as portfolios of European calls and puts on zero coupon bonds, respectively.
   
   (e) True or False. The market price of risk in a world that is forward risk neutral with respect to the dollar money market is zero.
   
   (f) True or False. Merton’s structural model allows for multiple debt maturities through an implied volatility surface for the assets of the firm.
   
   (g) True or False. Credit default swaps at various tenors always give the same hazard rate for the underlying issuer.
   
   (h) True or False. The 99% CVaR of a portfolio is always at least as large in magnitude as the 99% VaR.
   
   (i) True or False. The only numerical methods of the big three we covered in class that may be used to price American features in an option are Trees and Finite Difference schemes.
   
   (j) True or False. Mean reversion in rates would produce an arbitrage if they were directly tradeable.
2. Consider a portfolio of options with two underlyings, $S_1$ and $S_2$. Assume there are 100,000 options written on $S_1$, each with a $\Delta$ of 0.50, and 200,000 options written on $S_2$ with a $\Delta$ of $-0.25$. Let the stock prices of $S_1$ and $S_2$ are $10$ and $15$, respectively.

(a) What is the dollar-$\Delta$ of the portfolio?

(b) If the CAPM-$\beta$ of $S_1$ is 0.75, and the CAPM-$\beta$ of $S_2$ is 1.5, how many shares of the reference index would you need to buy or sell to be approximately $\Delta$ neutral if the index trades at $500$?

(c) Suppose the index has an annualized volatility of 25%. Using your work above, determine the 1 day 95% VaR for the option portfolio when it is not $\Delta$ hedged? You may ignore idiosyncratic volatility.

(d) Using the same work and assumptions, what is the 1 day 95% VaR when the portfolio is $\Delta$ hedged with the index?
3. Recall that the Ho-Lee model is given by

\[ dr = \theta(t)dt + \sigma dZ. \]

Let \( P(t, T) \) be the price of a zero coupon bond at time \( t \) which matures at time \( T \). We may write this as \( P(r(t), T) \).

(a) Using Ito, write down the dynamics of \( dP \) in terms of \( \frac{\partial P}{\partial r}, \frac{\partial P}{\partial t}, \) and \( \frac{\partial^2 P}{\partial r^2} \).

(b) We know that under Ho-Lee, \( P(r, t) = A(t, T)e^{-r(t)(T-t)} \) for some analytic \( A(t, T) \). What is \( \frac{\partial P}{\partial r} \) in terms of \( P \)?

(c) Using the last two results, what is \( \sigma_P \)?
4. Suppose a three year zero coupon bond is trading at $90 and a five year zero coupon bond is trading at $80, both with par value of $100.

   (a) What is the two-year forward rate for the period beginning in three years, \( R(0, 3, 5) \)?

   (b) What is the three year forward price of the five year zero coupon bond?

   (c) Use Black’s model to price a three year European call option on the five year bond with strike price of $85 when the volatility of the forward bond price is 25%. (You don’t need a final number here, but show enough work that one could be calculated exactly.)

5. Let Security \( X \) be positively dependent on both oil and gold, having market prices of risk of 0.4 and 0.6, respectively. We showed in class that

\[
dX = X\mu_X \, dt + X(\sigma_o dZ_o + \sigma_g dZ_g).
\]

   (a) When gold risk is hedged out from \( X \), the volatility of \( X \) is 20%, and when oil risk is hedged out, the volatility is 10%. What are \( \sigma_o \) and \( \sigma_g \)?

   (b) What is the excess return over the risk free rate for \( X \) if gold risk has been hedged out?
6. Recall that in the Hull-White model that the price of a zero coupon bond is given by

\[ P(t, T) = A(t, T)e^{-B(t, T)r(t)} \]

with

\[ B(t, T) = \frac{1 - e^{-a(T-t)}}{a}. \]

Let \( a = 0.4 \) and assume the bond considered below has a five year maturity with price $105 as of today, \( t = 0 \).

(a) We defined the alternative duration measure, \( \hat{D} \) by the relationship \( \frac{\partial P}{\partial r} = -\hat{D}P \). What is the alternative duration for the bond we are considering?

(b) Use the Taylor approximation \( \Delta P \approx -\hat{D}P \Delta r \) to approximate the change in bond price under a 10 bps change in rates.

(c) We may similarly define an alternative convexity measure, \( \hat{C} \) by the relationship \( \frac{\partial^2 P}{\partial r^2} = \hat{C}P \). What is the alternative convexity for this bond?

(d) Generalize the Taylor approximation above to account for alternative convexity. What is the approximate change in bond price under a 10 bps change in rates?
7. Suppose $f_1$ and $f_2$ follow $df_i = f_i \mu_i dt + f_i \sigma_i dZ$ for $i = 1, 2$. If $F = \frac{f_1}{f_2}$, what is $dF$ in a world that is forward risk neutral with respect to $f_2$?

8. If the spread of a 5 year CDS is 300 bps and the recovery rate of the reference bond is 40, what is an approximate implied probability of default over one year?

9. Suppose a stock is trading at $65, and you have used an implicit finite difference scheme to value a one year option, with a portion of the grid given by

<table>
<thead>
<tr>
<th>$S$ \ $\tau$</th>
<th>1</th>
<th>0.98</th>
</tr>
</thead>
<tbody>
<tr>
<td>66</td>
<td>0.47</td>
<td>0.39</td>
</tr>
<tr>
<td>65</td>
<td>0.82</td>
<td>0.71</td>
</tr>
<tr>
<td>64</td>
<td>1.42</td>
<td>1.27</td>
</tr>
</tbody>
</table>

where $\tau$ denotes the time to maturity in years.

(a) What is $\Delta$?

(b) What is $\Theta$?

(c) What is $\Gamma$?
Bonus Question ($+\infty$ points): Name an animal with more than one heart.

Use the rest for scratch