This page contains the answers only to the questions I wrote on the board in class today. There may be typos in this answer sheet, so if your answer differs from mine, you should check the solutions to verify which one is right. Please e-mail me if you do see a typo, and I will try to get back to you before tomorrow. Also note that I may have incorrectly stated in class that the phase angle for the last problem is $\pi/8$. The answer should be $\pi/4$, and you can check this with a calculator by graphing $6\cos(2t) + 6\sin(2t) - 6\sqrt{2}\cos(2t - \pi/4)$. You will get that the resulting graph is 0. Remember that I do not know what is going to be on the test tomorrow, so I suggest that you look over all of your notes, the sections in the book we covered, the review sheet, and homework in preparation for the test.

Problem 1. Solve the system

\[
\begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix} = \begin{bmatrix}
3 & -2 \\
5 & 5
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix}, \quad \begin{bmatrix}
x(0) \\
y(0)
\end{bmatrix} = \begin{bmatrix}
-2 \\
3
\end{bmatrix}.
\]

Answer.

\[
\begin{bmatrix}
x \\
y
\end{bmatrix} = \begin{bmatrix}
-2e^{4t}\cos(3t) - \frac{4}{3}e^{4t}\sin(3t) \\
3e^{4t}\cos(3t) - \frac{7}{3}e^{4t}\sin(3t)
\end{bmatrix}.
\]

Problem 2. Solve the system

\[
\begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix} = \begin{bmatrix}
5 & 1 \\
2 & 4
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix}, \quad \begin{bmatrix}
x(0) \\
y(0)
\end{bmatrix} = \begin{bmatrix}
1 \\
0
\end{bmatrix}.
\]

Answer.
Problem 3. Use the method of undetermined coefficients to solve

\[ y'' - 8y' + 15y = 26e^{4t}, \quad y(0) = 1, \quad y'(0) = 2. \]

Answer. \(12.5e^{5t} + 14.5e^{3t} - 26e^{4t}.\)

Problem 4. Solve

\[ y' = 4y^2 - 16y - 48 = 4(y - 6)(y + 2), \quad y(0) = 2. \]

Answer.

\[ y = \frac{6 - 2e^{32x}}{1 + e^{32x}}. \]

Problem 5. Solve

\[ y' + x^2 y = 5e^{-x^3/3}, \quad y(0) = 1. \]

Answer. \( y = 5xe^{-x^3/3} + e^{-x^3/3}.\)

Problem 6. Use the factoring operator method to find the general solution of

\[ y'' - 5y' - 14y = e^x. \]

Answer. \( y = -\frac{1}{18}e^x + C_1 e^{7x} + C_2 e^{-2x}.\)

Problems 7 and 8. Determine whether the following are underdamped, overdamped, critically damped, or simple harmonic. Then, find the general solution.

\[ y'' + 6y' + 9y = 0, \]
Answer. Critically Damped. \( y = C_1 e^{-3t} + C_2 t e^{-3t}. \)

\[ y'' + 7y = 0. \]

Answer. Simple harmonic. \( y = C_1 \cos(\sqrt{7}x) + C_2 \sin(\sqrt{7}x). \)

Problems 9 and 10. Write a trial solution with undetermined coefficients for a particular solution of the following:
\[ y'' = 5t + 6, \]

Answer. \( y_p = At^3 + Bt^2. \)

\[ y'' + y' + y = x \cos(x) - e^x. \]

Answer. \( y_p = Ax \cos(x) + Bx \sin(x) + C \cos(x) + D \sin(x) + E e^x. \)

Problem 11. Find a differential equation of the form \( ay'' + by' + cy = 0 \) that is satisfied by the following:
\[ e^{-6t} \cos(4t). \]

Answer. \( y'' + 12y' + 52y = 0. \)

Problem 12. Convert \( 6 \cos(2t) + 6 \sin(2t) \) to phase angle form. Answer. \( 6 \sqrt{2} \cos(2t - \pi/4) \)