(1) (5 Points) Does $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ converge? Justify your answer. [This is taken from the Integral Test homework set (11.3)]

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

compares to $\sum_{n=1}^{\infty} \frac{1}{n}$

$$\lim_{n \to \infty} \frac{n}{n^2+1} \cdot \frac{1}{n} = \lim_{n \to \infty} \frac{n^2}{n^2+1} = 1$$

Since $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, (D) so does $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$.

(2) (5 Points) Does $\sum_{n=0}^{\infty} \frac{n}{2n^3+1}$ converge? Justify your answer. [This is taken from the Comparison Tests homework set (11.4)]

$$\sum_{n=0}^{\infty} \frac{n}{2n^3+1} < \sum_{n=0}^{\infty} \frac{n}{2n^3} = \frac{1}{2} \sum_{n=0}^{\infty} \frac{1}{n^2}$$

which is convergent by the $p$-series test.

So by the comparison test, $\sum_{n=0}^{\infty} \frac{n}{2n^3+1}$ converges.