1. A container in the shape of a paraboloid (a parabola revolved around an axis going through the vertex of the parabola and the directrix, so every horizontal slice, or cross section, is a circle.) is being filled with a liquid. The paraboloid is 2 feet in height and at its maximum width is 14 inches wide. Find: (Do NOT use information from one part in another part!)

(a) the rate at which the height is changing when the liquid is 10 inches high if the volume is changing at 0.125 $ft^3/s$

(b) the rate at which the radius is changing when the liquid is at a radius of 6 inches if the volume is changing at 0.125 $ft^3/s$

(c) the rate at which the volume is changing when the liquid is 12 inches high and at a radius of 6 inches if the radius is changing at 0.5 $ft/s$ and the height is changing at 0.25 $ft/s$

$$V_{\text{paraboloid}} = \frac{1}{2} \pi r^2 h$$
2. Consider the function

\[ f(x) = e^x \sin(x) \text{ for } -\pi \leq x \leq \pi \]

State the intervals of increase and decrease. State and justify any extrema based on the intervals of increase and decrease only. (Hint: Use the First Derivative Test.)
3. Determine the intervals of concavity, points of inflection and classify (with justification) any extrema via the Second Derivative Test of the function

\[ f(x) = x^6 - 6x^4 \text{ for } -5 \leq x \leq 5 \]
4. Compute: (Note: \( n! = n(n-1)(n-2)\cdots(2)(1) \))

\[
\lim_{x \to 0} \frac{e^{5x} - 5x - 1}{\sin(x) - x}
\]

\[
\lim_{x \to \infty} \frac{x^5 e^{-x}}{5!}
\]
5. Bruno is on a budget and wants to build a storage shed with a square base for 1500 dollars. After taking a visit to Cheap Skates ’R Us, he finds out that for the walls it will cost him about 3 dollars per square foot, the floor will be 4 dollars per square foot and the ceiling will cost him 6 dollars per square foot. What dimensions of the shed would yield the greatest volume if there is also 100 dollars in miscellaneous expenses for the shed?
6. Sketch the graph of the continuous function \( f(x) \) and classify the extrema and points of inflection (with justification) if:

(a) \( f'(-5) = 0, f'(-2) = 0, f'(0) = DNE, f'(3) = 0, f'(6) = DNE \)

(b) \( f''(-4) = 0, f''(0) = DNE, f''(1) = 0, f''(6) = DNE \)

(c) \( f'(x) < 0 \) on \(-2 < x < 0, x > 6, 0 < x < 3, \) and \( x < -5\). \( f'(x) > 0 \) on \( 3 < x < 6, -5 < x < -2 \)

(d) \( f''(x) > 0 \) on \( x < -4, 1 < x < 6, x > 6 \). \( f''(x) < 0 \) on \( -4 < x < 0, 0 < x < 1 \)
7. Prove via tangent lines that the following function is concave up at the point $(6, -18)$ if

\[ f(x) = x^2 - 10x + 6 \]
CHALLENGE:
8. Suppose a tank in the shape of a regular octahedron (two right square pyramids with the square bases glued together where all the side lengths are of equal length) is full of liquid and emptying into a tank of equal volume that is in the shape of a regular tetrahedron (a right triangular pyramid with all side lengths equal). Suppose all the liquid goes directly from the octahedron to the tetrahedron at some rate $r$. Find

(a) the rate at which the height of the liquid in the octahedron is decreasing (there are two cases to consider!)

(b) the rate at which the height of the liquid in the tetrahedron is increasing

$$V_{pyramid} = \frac{1}{3} A_{Base} h$$

**Note: This is a theory problem – carefully define your variables and work “in general.”
CHALLENGE:
9. Engineers have made a new material – rubber paper! It looks and feels just like normal paper but stretches in every direction a bit before it breaks. It is also a fact that a torus (looks like a doughnut) can be made from a piece of paper if you are careful enough. (First make it into a cylinder without a top or bottom then glue the open ends of the cylinder together.) Suppose you have a sheet of this rubber paper. It is in the shape of a rectangle and has a surface area of $A$. When stretched to its limit it can double its surface area. What is the maximal volume of a torus of this rubber paper when the paper is not stretched? What is the maximal volume of a torus of the rubber paper when it is stretched?

$$SA_{cylinder} = 2\pi rh$$
$$V_{cylinder} = \pi r^2 h$$

**Note: This is a theory problem – carefully define your variables and work “in general.”