MATH 3283W
Quiz 5
Tuesday 4 April 2017

Remember, your work will be graded on the quality of your writing as well as on the validity of the mathematics. Writing score: 5 points. The quiz is worth 20 points total.

1. (5 points) Complete the following definition: We say that a sequence \( \{s_n\} \) converges to the real number \( s \) if …

   for every \( \varepsilon > 0 \), there exists an \( N \in \mathbb{N} \) such that
   \[ |s_n - s| < \varepsilon \quad \text{whenever} \quad n > N. \]

2. (10 points) Find the limit of the sequence \( \{s_n\} \) with
   \[ s_n = \frac{3n^2 + 2n}{2n^2 - 3n}. \]

   Justify your answer using the definition of convergence directly. Do not use any theorems that have been proven in class or in the textbook. The sequence is recopied on the back, in case you want to use the back for scratch work. Write your formal justification here on the front.

   The limit of the sequence is the ratio of top degree terms, when the degrees are equal. So we guess \( \lim s_n = \frac{3}{2} \). To prove it,

   consider
   \[ |s_n - s| = \left| \frac{3n^2 + 2n}{2n^2 - 3n} - \frac{3}{2} \right| = \left| \frac{6n^2 + 4n - 3(2n^2 - 3n)}{2(2n^2 - 3n)} \right| \]

   \[ = \left| \frac{13n}{2(2n^2 - 3n)} \right| \]

   We would like to show this quantity is small (\(<\) given \( \varepsilon \)) when \( n \) sufficiently large (\( > N(\varepsilon) \))

   Scratchwork: \( n^2 < 2n^2 - 3n \) \( \iff \) \( 3n < n^2 \), which holds if \( n > 4 \).

   thus if \( n > 4 \), \( \left| \frac{13n}{2(2n^2 - 3n)} \right| = \frac{13n}{2(2n^2 - 3n)} < \frac{13n}{2 \cdot n^2} = \frac{13}{2} \cdot \frac{1}{n}. \)

   want \( \frac{13}{2} \cdot \frac{1}{n} < \varepsilon \), so \( \frac{1}{n} < \frac{2\varepsilon}{13} \) i.e. \( n > \frac{13}{2\varepsilon} \).

   **pf:** Given \( \varepsilon > 0 \), Set \( N(\varepsilon) = \max \left\{ 4, \frac{13}{2\varepsilon} \right\} \). Then by scratchwork, if \( n > N(\varepsilon) \), then \( |s_n - s| = \left| \frac{13n}{2(2n^2 - 3n)} \right| < \frac{13}{2n} < \varepsilon \) since \( n > \frac{13}{2\varepsilon} \) (in last step).