Proof: If $f$ has no zero in $\mathbb{D}$, $f$ is analytic in $\mathbb{D}$ and continuous on its boundary. According to Maximum Modulus Principle, $\frac{1}{f}$ attains its maximum on $\partial \mathbb{D}$, which means $1$ attains its maximum minimum on $\partial \mathbb{D}$.

If $f$ has a zero in $\mathbb{D}$, there is nothing to prove since $1/f$ will attain the minimum on that zero.

Proof: If $f$ has no zero in $\mathbb{D}$, $f$ is analytic in $\mathbb{D}$ and continuous on its boundary. According to Maximum Modulus Principle, $\frac{1}{f}$ attains its maximum on $\partial \mathbb{D}$.

Since $|f| = c$ on the boundary of $\mathbb{D}$, $\frac{1}{f} = \frac{1}{c}$ on $\partial \mathbb{D}$, and $\frac{1}{f} \leq \frac{1}{c}$ for $2e^{\mathbb{S}}$, that's $|f| \geq \frac{1}{c}$.

By the same way, for $f(z)$, we get $|f| \leq c$ for $\forall z \in \mathbb{D}$.

Therefore $|f| = c$ for $\forall z \in \mathbb{S}$.

Since $f$ is analytic and bounded, according to Liouville's Thm, $f$ must be a constant.

If $f$ has a zero in $\mathbb{D}$, there is nothing to prove.
3. Proof: For \( c \in (0,1) \), there exists another constant \( D \), s.t. \( 0 < D < 1 \).

For \( \forall \varepsilon > 0 \), the function
\[
F_{\varepsilon}(z) = \frac{f(z)}{e^{\varepsilon z}}
\]
is bounded by 1 on the edges of the half-strip.

And \( F_{\varepsilon}(z) \to 0 \) uniformly for \( \text{Im}(z) \) as \( \text{Re}(z) \to \infty \), \( z \in D \).

Thus, we can find a \( T(\varepsilon) \) on the edge of a rectangle
\[
0 \leq \text{Re}(z) \leq T(\varepsilon), \quad -\frac{T}{3} \leq \text{Im}(z) \leq \frac{T}{3}
\]
According to Maximum Modulus Principle
\[
|F_{\varepsilon}(z)| \leq 1 \quad \text{on the rectangle.}
\]
That's if \( |f(z)| \leq e^{\varepsilon z} \) in the interior of the rectangle.

Let \( \varepsilon \to 0 \), then \( T(\varepsilon) \to \infty \), and
\[
|f(z)| \leq 1
\]

4. Proof: If \( \gamma \) is homotopic to \( \gamma_a \), there exists a continuous function \( \Gamma_a(s, t) \), s.t.
\[
\begin{align*}
\Gamma_a(s, 0) &= \gamma_a(s) \equiv a \\
\Gamma_a(1, t) &= \gamma(t) \\
\Gamma_a(s, t) &= \gamma(s, t) \\
\end{align*}
\]
Let \( \Gamma_b(s, t) = \Gamma_a(s, t) + (1-t)(b-a) \)
Therefore \( \gamma(0) = e^{i\theta} \cdot 0 = 0 \). Then \( \gamma(t) \) is not closed. Since

For a closed curve \( \gamma(0) = e^{i\theta} \cdot 0 = \gamma(0) \),

\[
\gamma(t) = e^{i\theta} (\cos(\theta t), \sin(\theta t))
\]

Let \( \gamma(t) = \frac{1}{2} \). Then

\[
\gamma(t) = e^{i\theta} (\cos(\theta t), \sin(\theta t))
\]

\[
\gamma(t) = e^{i\theta} (\cos(\theta t), \sin(\theta t))
\]

For a closed curve \( \gamma(0) = e^{i\theta} (\cos(\theta t), \sin(\theta t)) \)

\[
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\]

Therefore \( \Gamma \) is homotopic to the constant curve.
\[
\int f(z) \, dz = \int \frac{dz}{z^2} = \begin{cases} 
\frac{-1}{z} & \text{if } z \neq 0 \\
\pi i & \text{if } z = 0
\end{cases}
\]

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\[
\int f(z) \, dz = \int f(z) \, dz
\]

\[
(2) \quad \frac{d}{dz} f(z) = \frac{d}{dz} f(z)
\]

\[
(3) \quad \frac{d}{dz} f(z) = \frac{d}{dz} f(z)
\]

\[
\int f(z) \, dz = \int f(z) \, dz
\]