1 Math 8401: Review Assignment 1

The purpose of this assignment is to review background material that will be used in the course.

1. Find the Fourier series of the following functions:
   1. \( f(x) = x, \ x \in [-\pi, \pi] \).
   2. \( f(x) = x \sin x, \ x \in [-\pi, \pi] \).
   3. \( f(x) = |x|, \ x \in [-\pi, \pi] \).

2. Consider a bar of length one. The function \( u(t, x) \) represents the temperature at the point \( x \) in the bar at time \( t \). Find a series solution to the initial-boundary value problem for the heat equation
   \[ u_t = u_{xx}, \ x \in (0,1), \]
   when one end of the bar is held at 0 degrees and the other end is insulated. The initial distribution of the temperature is given as \( u(0, x) = (x - 1)^2 - 1 \).

3. Solve the boundary value problem for Laplace’s equation
   \[ u_{xx} + u_{yy} = 0, \ x \in (0, \pi), \ y \in (0, \pi), \]
   \[ u(0, y) = u(\pi, y) = u(x, \pi) = 0, \]
   \[ u(x, 0) = \sin \pi x. \]

References.
- *A First Course in Partial Differential Equations* by Hans Weinberger.
- *Introduction to Partial Differential Equations* by Peter Olver.
- *Introduction to Partial Differential Equations* by Matthew Coleman.

All the references are available at the Library.