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Seven Decades of Mathematics and Mechanics

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On the occasion of Jerald LaVerne Ericksen's 90th birthday, a group of mathematicians and scientists came together in Eugene, Oregon, last October to celebrate accomplishments in the field of mathematics and mechanics. The workshop, entitled "Mathematics and Mechanics in the 22nd Century: Seven Decades and Counting," spanned the seven decades of Ericksen's research career and offered a view of future research prospects.[1]

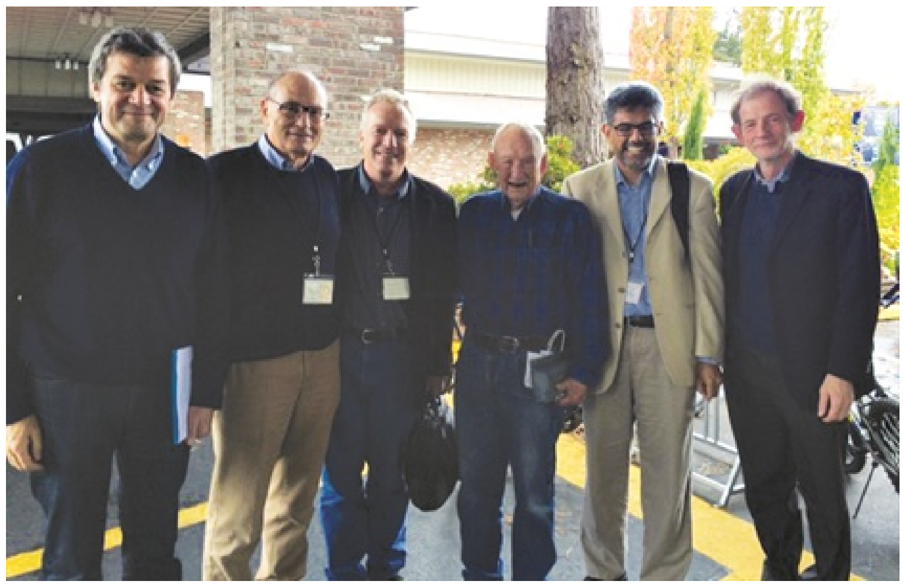
Maria-Carme Calderer, Jerry Ericksen, and Irene Fonseca at the "Mathematics and Mechanics in the 22nd Century: Seven Decades and Counting" workshop held last October. Photo credit: David Kinderlehrer.

In the late 1940s, researchers recognized both the inadequacy in existing theory of the description of complex materials and the need for new mathematical concepts and tools. This led to the emergence of modern applied mathematics, in which new concepts in geometry, analysis, partial differential equations, and numerical, computational and stochastic mathematics develop synergistically with molecular and continuum mechanics. The concurrent rise of the fields of materials science and polymer chemistry also contributed to this synergy.

Due to the works of Ericksen and Ronald Rivlin in the 1950s and 1960s, nonlinear elasticity attracted the attention of the mathematics community. Ericksen's research offered a deep understanding of how invariance principles served to identify special solutions. Two landmark papers from his work during this period deliver explicit classifications of all deformations possible in every incompressible and compressible nonlinear elastic material, regardless of the specific form of the free energy function [5, 7]. Ericksen's work established a fertile mathematical link between the calculus of variations and the stability of materials. But researchers soon realized that the available theory of the direct method at that time was inadequate.

Researchers such as John Ball and Stuart Antman closed this gap in the 1970s, which led to a resurgence of work in the calculus of variations that continues to this day. The study of multiscale problems called for better qualitative methods, and Ennio De Giorgi's [3, 4] method of-convergence assumed a central role. Due to this body of research, we now have a rigorous understanding of the formerly disparate theories of mechanics; this insight increasingly underlies the design and discovery of both new materials and new device concepts [13]. Additionally, research on time-dependent problems led to a deeper understanding of nonlinear hyperbolic partial differential equations, notably represented by the contributions of Constantine Dafermos. Both frequent cross fertilization with closely-related lines of research and a major effort to extend multiscale methods to the atomic scale led to the body of work now associated with the mathematics of materials science.

A vital branch of this research seeks to develop rigorous methods for atomic-to-continuum scales, and encompasses rigorous spatial or temporal averaging, improvement or potential replacement of density functional theory, better understanding of statistical mechanics for solid crystals, and inroads into non-equilibrium statistical mechanics. Given the profound successes of continuum-to-continuum methods, nobody seemed to realize how difficult these atomic-to-continuum problems would be. Nevertheless, they are incredibly important, as they fundamentally underlie the relation between the composition of the material and its properties.



Lev Truskinovsky (Ecole Polytechnique), Cesare Davini (University of Udine), Richard James (University of Minnesota), Jerry Ericksen (University of Minnesota, emeritus), Kaushik Bhattacharya (California Institute of Technology), and John Ball (University of Oxford).

Until the late 1950s, when Ericksen began working on liquid crystals, research in the field was largely confined to a small subset of the chemistry community. At about the same time, the emergence of the Orsay Group, led by Pierre Giles de Gennes (1991 Nobel Prize in Physics) marked the beginning of the study of "soft matter" as a unified field. Ericksen came across liquid crystals while detecting deficiencies in the theories of anisotropic fluids [1]. His article on anisotropic fluids [6] initiated two decades of work by himself, Frank Leslie, and their collaborators, giving rise to the renowned flow theory of liquid crystals.

The equilibrium theory of liquid crystals, formulated decades earlier by Zocher [14], Oseen [11], and Frank [8], prompted an independent path of research within the framework of the direct methods of calculus of variations. Ericksen's arrival at the University of Minnesota forged a link between the analysis and liquid crystal research communities. Ericksen started a graduate course on liquid crystals, through which he met senior mathematician David Kinderlehrer, who became involved in the research himself. Mathematical research on the Ericksen-Leslie equations of liquid crystal flow reemerged, with many challenging problems remaining open to this day. These equations naturally inherit the analytic difficulties of the Navier-Stokes equations and those of the Oseen-Frank energy, in connection with the presence of defects.

A main paradigm in applications of liquid crystals is the ability to control macroscopic size regions of uniform molecular alignment. In display devices, electromagnetic fields achieve this alignment, with the Oseen-Frank energy as the industrial workhorse. The current and next generation of liquid crystal research aims toward the 'topological control' of the systems, with liquid crystal colloids playing a prominent role [2, 12]. These colloids, either in the form of dispersed particles in a liquid crystal or as chromonic phases where the liquid crystal is embedded in an isotropic matrix, offer amazingly rich and novel defect structures. Applying electromagnetic fields to such systems yields new mechanisms to control particle motion and aggregation, with relevant consequences in biological research [10]. Likewise, development of composite materials via patterned liquid crystal substrates is creating a new path in metamaterials research. Furthermore, the discovery of new types of liquid crystals with efficient conversion factors from mechanical to electrical energy currently has a major impact on organic electronics research, including semiconductors and solar cells [9]. These advances open up new synergistic opportunities for the mathematical community to drive technological invention.

[1] The organizers wish to acknowledge financial support from the National Science Foundation and other sponsors. See here for details.

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