Course description: The purpose of this course is to study mathematical methods for modeling materials behavior. It combines the use of physical principles with the analysis of the ensuing mathematical problems. The course starts with the derivation of the equations of balance of mass, linear momentum and energy of a continuum system of particles. From the integral forms of such laws, we obtain their formulation as systems of partial differential equations. Energy dissipation inequalities on the solutions stem from the requirement that the system satisfy the Second Law of Thermodynamics. These inequalities provide a necessary tool for the well-posedness of the governing system of equations. However, these alone are in generally undetermined, since there are too many unknown fields. The required information is provided in the form of constitutive equations that describe families of material behavior, such as viscoelastic fluids and elastic solids. We will place special emphasis on elastic solids, by analyzing boundary value problems modeling equilibrium and the corresponding variational methods. We will examine dimensional reduction techniques, based on the method of Gamma convergence to explore the derivation of rod and plate models, from the three-dimensional setting. We will then address, time-dependent problems for elastic bodies and viscoelastic fluids, leading to models of polyelectrolyte gels. These are systems that combine elastic networks and fluids, with charged ions, and are relevant to the modeling of biological structures and devices. Models of these systems include the coupling of the mechanical equations of the gels with the Nears-Plank equations of transport and diffusion of charged particles.

In applications to biology, elastic systems of interest are highly anisotropic. We will present some new modeling techniques that employ concept of liquid crystal elastomers to model systems of fibers.

Textbooks: The following is a list of some of the books that will be used in the course:

Assignments and Examinations: There will be approximately 4 homework assignments, two midterm and the final examinations. The grade of the course will be based upon a weighted average of homework and examinations:

Homework: 25 %;
Midterm Examination I (Wednesday, February 22, 9:05–9:55 am): 20 %;
Midterm Examination II (Wednesday, April 7, 9:05–9:55 am): 20 %;
Final Examination (As scheduled in the University calendar): 35 %.

The list of topics includes:

- Balance laws and Thermodynamics;
- Lagrangian and Eulerian coordinates;
- Constitutive Laws;
- Material frame-indifference and symmetry; isotropy;
- Equilibrium equations of elasticity and the principle of virtual work;
- Energy minimization: a model problem;
- Elastic materials and constitutive equations; hyperelasticity
- Weak topology and weak convergence; properties of determinants of matrices;
- Linearized elasticity;
- Newtonian Fluids and the Navier-Stokes Equations;
- Dissipation inequalities;
- Method of Gamma-convergence and dimensional reduction: rods and plates.
- Anisotropic elasticity and modeling of fibers: liquid crystal elastomers
- Polyelectrolyte gels