Read Sections 11.1 (skip Theorem 11.2) and 9.1 (up to Theorem 9.5) in your textbook.

1. Show that the Klein four-group $\mathcal{V}$ is not isomorphic to $\mathbb{Z}_4$.

2. Prove that $\mathbb{Z}_7^\times \cong \mathbb{Z}_6$. (It is crucial to remember that we multiply in $\mathbb{Z}_7^\times = U(7)$ and add in $\mathbb{Z}_6$.)

3. a. Prove that $\mathbb{Z}_{12}^\times \cong \mathcal{V}$.
   
   b. Prove that $\mathbb{Z}_{15}^\times \cong \mathbb{Z}_{16}^\times \cong \mathbb{Z}_{20}^\times$. What about $\mathbb{Z}_{24}^\times$?

4. Show that $\phi: \mathbb{R} \rightarrow \mathbb{C}^\times$ given by $\phi(t) = \text{cis}(2\pi t)$ is a homomorphism. Then describe its kernel and image.

5. Let $a \in G$ be fixed, and define $\phi: G \rightarrow G$ by $\phi(x) = axa^{-1}$. Prove that $\phi$ is a homomorphism. Under what circumstances is $\phi$ an isomorphism?

6. Let $\zeta = \text{cis} \left( \frac{2\pi}{n} \right)$. Prove that the dihedral group $D_n$ is isomorphic to the subgroup of $GL_2(\mathbb{C})$ obtained by taking all products of the two matrices $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} \zeta & 0 \\ 0 & \zeta \end{bmatrix}$ and their inverses. (In other words, by taking the subgroup generated by these two matrix elements.)

Through the course of this assignment, I have followed the guidelines of the University of Minnesota Student Conduct Code.

Signed: ____________________________