

Name: \_\_\_\_\_

Problem Set 2  
Math 4281, Fall 2013  
Due: Friday, September 20

Read Sections 3.1 (thru top of p. 40 only), 6.3 (skip Theorems 6.11 and 6.12), 16.5, and 1.2 in your textbook.

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Complete the following items, staple this page to the front of your work, and turn your assignment in at the beginning of class on Friday, September 20.

1. You have at your disposal arbitrarily many 4-cent stamps and 7-cent stamps. What are the postages you can pay? Show in particular that you can pay all postages greater than 17 cents.
2. Complete the following exercise from your textbook.  
p. 34 #29
3. Determine the last digit of  $3^{400}$ , then the last two digits. Determine the last digit of  $7^{99}$ .
4. Suppose  $p$  is prime and  $a$  and  $b$  are integers. Prove that if  $a^2 \equiv b^2 \pmod{p}$ , then  $a \equiv b \pmod{p}$  or  $a \equiv -b \pmod{p}$ .
5. Prove that if  $x^2 \equiv n \pmod{65}$  has a solution, then so does  $x^2 \equiv -n \pmod{65}$ .
6. Solve the following congruences:
  - a.  $6x + 3 \equiv 1 \pmod{10}$
  - b.  $15x \equiv 25 \pmod{35}$
  - c. Simultaneously:  $x \equiv 1 \pmod{4}$ ,  $x \equiv 7 \pmod{13}$
  - d. Simultaneously:  $x \equiv 11 \pmod{142}$ ,  $x \equiv 25 \pmod{86}$
7. Define a relation on  $\mathbb{R}$  as follows:  $x \sim y$  if and only if  $x - y$  is an integer. Prove that  $\sim$  is an equivalence relation and describe the set of equivalence classes.
8. Given a function  $f: S \rightarrow T$ , consider the following relation on  $S$ :  $x \sim y \Leftrightarrow f(x) = f(y)$ .
  - a. Prove that  $\sim$  is an equivalence relation.
  - b. Prove that if  $f$  maps onto  $T$ , then there is a one-to-one correspondence between the set of equivalence classes and  $T$ .

Through the course of this assignment, I have followed the guidelines of the University of Minnesota Student Conduct Code.
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Signed: \_\_\_\_\_