Name: _____

Problem Set 12
Math 4281, Fall 2016
Due: Wednesday, November 30

Complete the following items, staple this page to the front of your work, and turn in your assignment at the beginning of class on the due date. Remember to list your collaborators on each problem.

Permutation groups

1. Given the permutations $\sigma = (1\ 2\ 4)$, $\tau = (1\ 3)(2\ 4) \in S_4$, compute the following elements:
   a. σ^{-1} b. $\sigma\tau$ c. $\tau\sigma$ d. σ^2 e. $\sigma^2\tau$ f. $\sigma\tau\sigma^{-1}$ g. $\tau\sigma\tau^{-1}$

2. a. Prove that a k-cycle in S_n is an element of order k.
   b. Prove that when we represent a permutation as a product of disjoint cycles, the order of the permutation is the least common multiple of the lengths of these cycles.

3. Determine if $\sigma = (1\ 2)(1\ 3\ 4)(1\ 5\ 2), \tau = (1\ 2\ 4\ 3)(3\ 5\ 2\ 1) \in S_5$ are even or odd.

4. Prove that A_n contains an n-cycle if and only if n is odd.

Group homomorphisms and isomorphisms

5. Show that $\phi\colon \mathbb{R} \to \mathbb{C}^\times$ given by $\phi(t) = \operatorname{cis}(2\pi t)$ is a homomorphism. Show that $\mathbb{Z}$ is the kernel of ϕ and the unit circle in the complex plane is the image of ϕ.

6. Let $a \in G$ be fixed, and define $\phi\colon G \to G$ by $\phi(x) = axa^{-1}$. Prove that ϕ is a homomorphism. Under what circumstances is ϕ an isomorphism?

7. Let $\zeta = \operatorname{cis}\left(\frac{2\pi}{n}\right)$. Prove that the dihedral group D_n is isomorphic to the subgroup of $\mathrm{G}L_2(\mathbb{C})$ obtained by taking all products of the two matrices $\left[\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right]$ and $\left[\begin{smallmatrix} \zeta & 0 \\ 0 & \bar{\zeta} \end{smallmatrix}\right]$ and their inverses. (In other words, by taking the subgroup generated by these two matrix elements.)

Throughout the course of this assignment, I have followed the guidelines of the University of Minnesota Student Conduct Code.

Signed: _____