Name: _____

Problem Set 4
Math 4281, Fall 2016
Due: Wednesday, October 5

Complete the following items, staple this page to the front of your work, and turn in your assignment at the beginning of class on the due date. Remember to list your collaborators on each problem.

Equivalence relations

1. Define a relation on $\mathbb{R}$ as follows: $x \sim y$ if and only if $x - y$ is an integer. Prove that $\sim$ is an equivalence relation and describe the set of equivalence classes.

2. Given a function $f\colon S \to T$, consider the following relation on S: $x \sim y \Leftrightarrow f(x) = f(y)$.

   a. Prove that $\sim$ is an equivalence relation.

   b. Prove that if f maps onto T, then there is a one-to-one correspondence between the set of equivalence classes and T.

Rings, domains, and fields

3. Let $a, m \in \mathbb{Z}$ with $m > 0$.

   a. Prove that $\gcd(a, m) = 1$ if and only if $[a] = \overline{a} \in \mathbb{Z}_m$ is a unit.

   b. Prove that if $[a] = \overline{a} \in \mathbb{Z}_m$ is a zero divisor, then $\gcd(a, m) > 1$, and conversely, provided $m \nmid a$.

4. Determine if the set $R = \{a + b\sqrt[3]{3} \mid a, b \in \mathbb{Q}\}$ is a ring with respect to the usual operations of addition and multiplication. If so, is it also a field?

5. Give a criterion (beyond the definition) for a matrix in $M_2(\mathbb{Z})$ to be a unit. Also, give a complete list of the units in $M_2(\mathbb{Z}_2)$.

6. Show that $R = \{a + b\sqrt{3}i \mid a, b \in \mathbb{Z}, i^2 = -1\}$ is a ring.

Throughout the course of this assignment, I have followed the guidelines of the University of Minnesota Student Conduct Code.
Signed: _____